

# **NEAR INFRARED FLUORESCENCE MOLECULAR IMAGING FOR CANCER DIAGNOSTICS**

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# INTRODUCTION

Among the most important problems in medical imaging is the three-dimensional reconstruction of biostructures embedded into tissues.

- 👁️ Need for non-invasive and safe for the patients techniques,
- 👁️ Need for reconstruction accuracy and time efficacy.

Fluorescence imaging is advancing to a very essential tool for medical imaging and diagnosis, mainly for cancer diagnosis.

- ▶️ Progress in fluorescent probes technology and optical imaging modalities.
- ▶️ Data quantification and processing is now more realistic and possible.



# OUTLINE

The fluorescence phenomenon.

Fluorescence molecular imaging - the problems.

Image acquisition systems.

Light propagation models.

Conclusion.



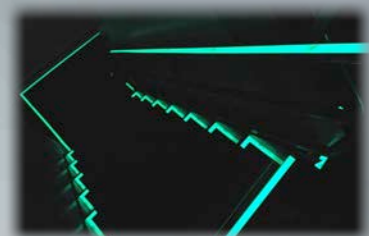
# LUMINESCENCE

Light emission from a medium, which is not due to high temperature, is called luminescence.

Since luminescence is actually energy leaving the medium, some kind of energy absorption should precede light emission...

...and thus we have:

- Electroluminescence (LEDs),  
*Substance excitation due to electric current.*
- Radioluminescence (emergency exit signs),  
*Substance excitation due to ionizing radiation.*
- Chemiluminescence (glow sticks) and  
*Substance excitation due to chemical reaction.*
- Photoluminescence (safety signs).  
*Substance excitation due to photon absorption.*



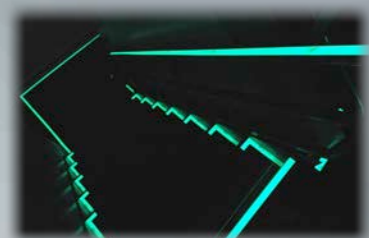
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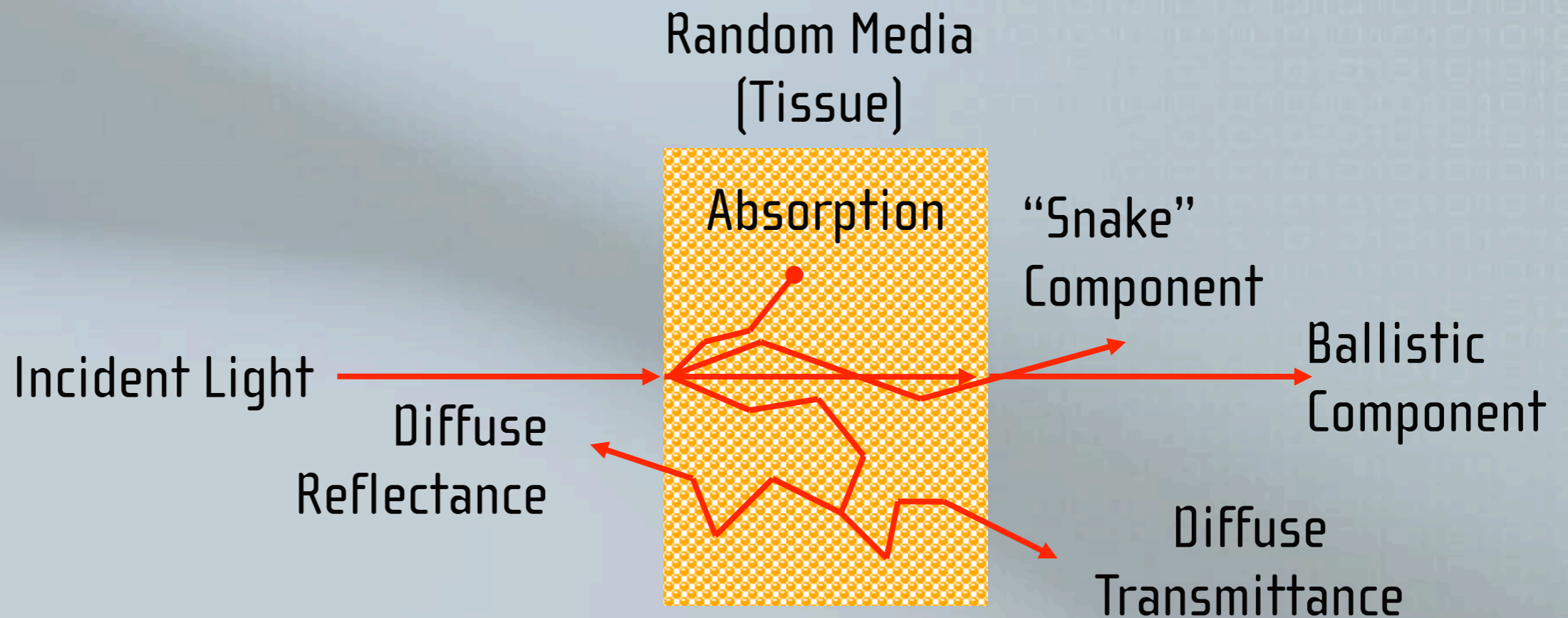
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- Photoluminescence (safety signs).  
*Substance excitation due to photon absorption*



# LIGHT PROPAGATION IN RANDOM MEDIA



# LIGHT PROPAGATION IN RANDOM

## MEDIA (CONT.)

- **Absorption coefficient:** The inverse quantity corresponds to the mean depth a photon travels inside the medium before it is absorbed.

Beer's Law:

$$I(x) = I_0 \cdot e^{-\mu_a \cdot x}$$

- **Scattering coefficient:** The inverse quantity corresponds to the mean depth a photon travels inside the medium before it is scattered.

Equivalent to Beer's Law:

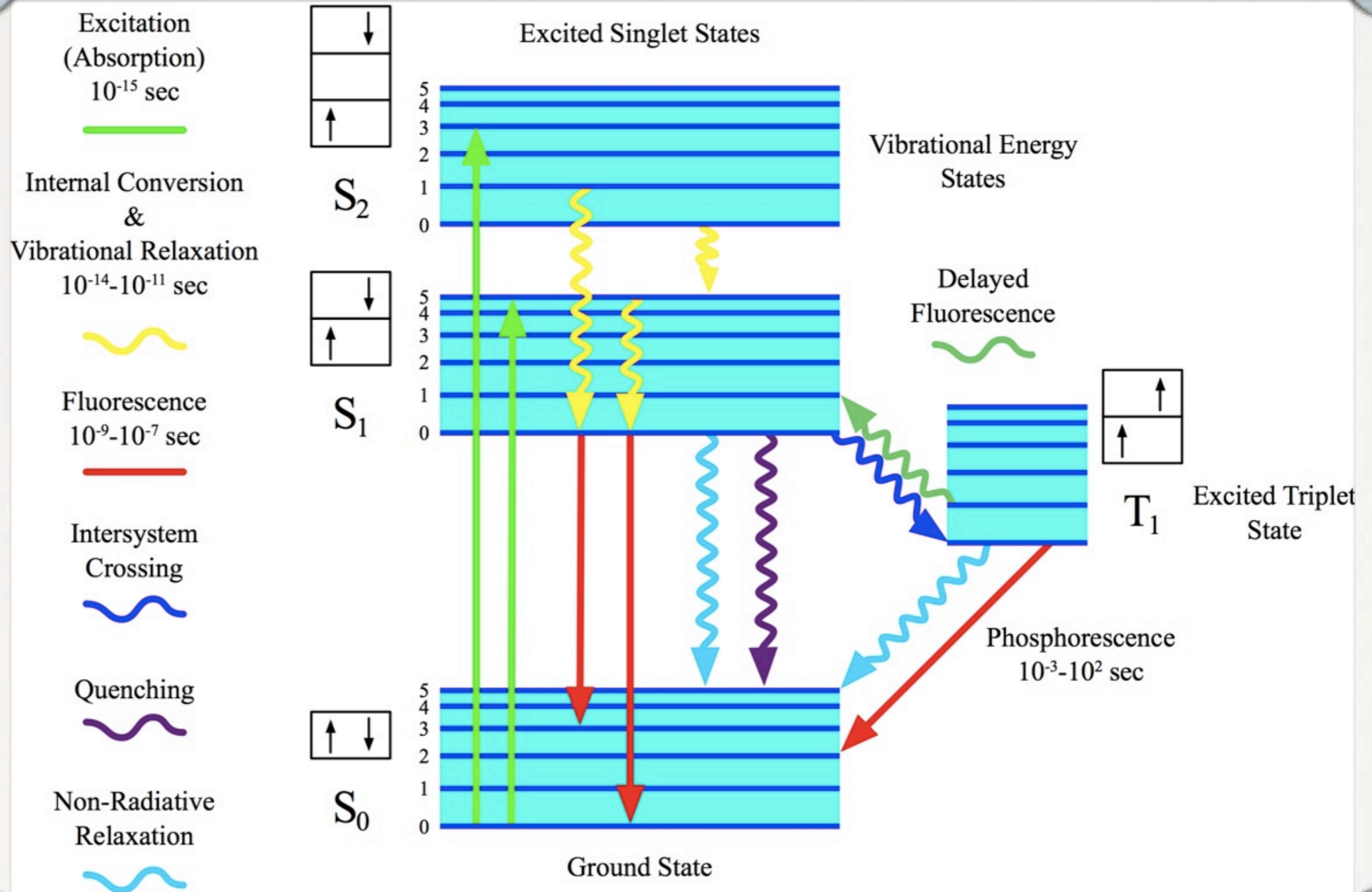
$$I(x) = I_0 \cdot e^{-\mu_s \cdot x}$$

- **Penetration depth:** The depth of the medium where light intensity equals to 37% (1/e) of the incident light intensity. This quantity equals to the inverse of the attenuation coefficient  $\mu_t = \mu_a + \mu_s$ .

Lambert Law:

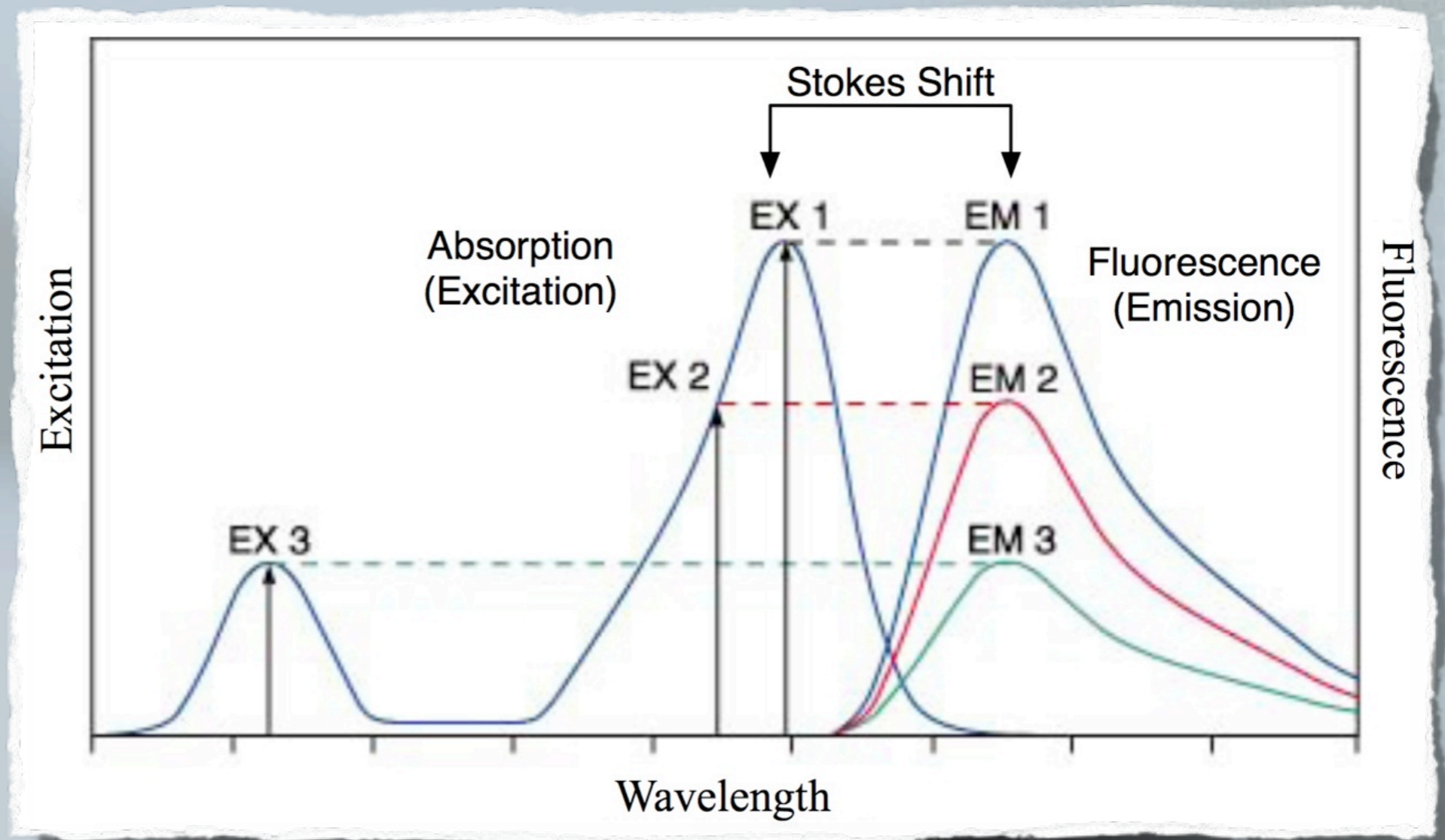
$$I(x) = I_0 \cdot e^{-\mu_t \cdot x}$$

# JABLONSKI DIAGRAM





# STOKES SHIFT

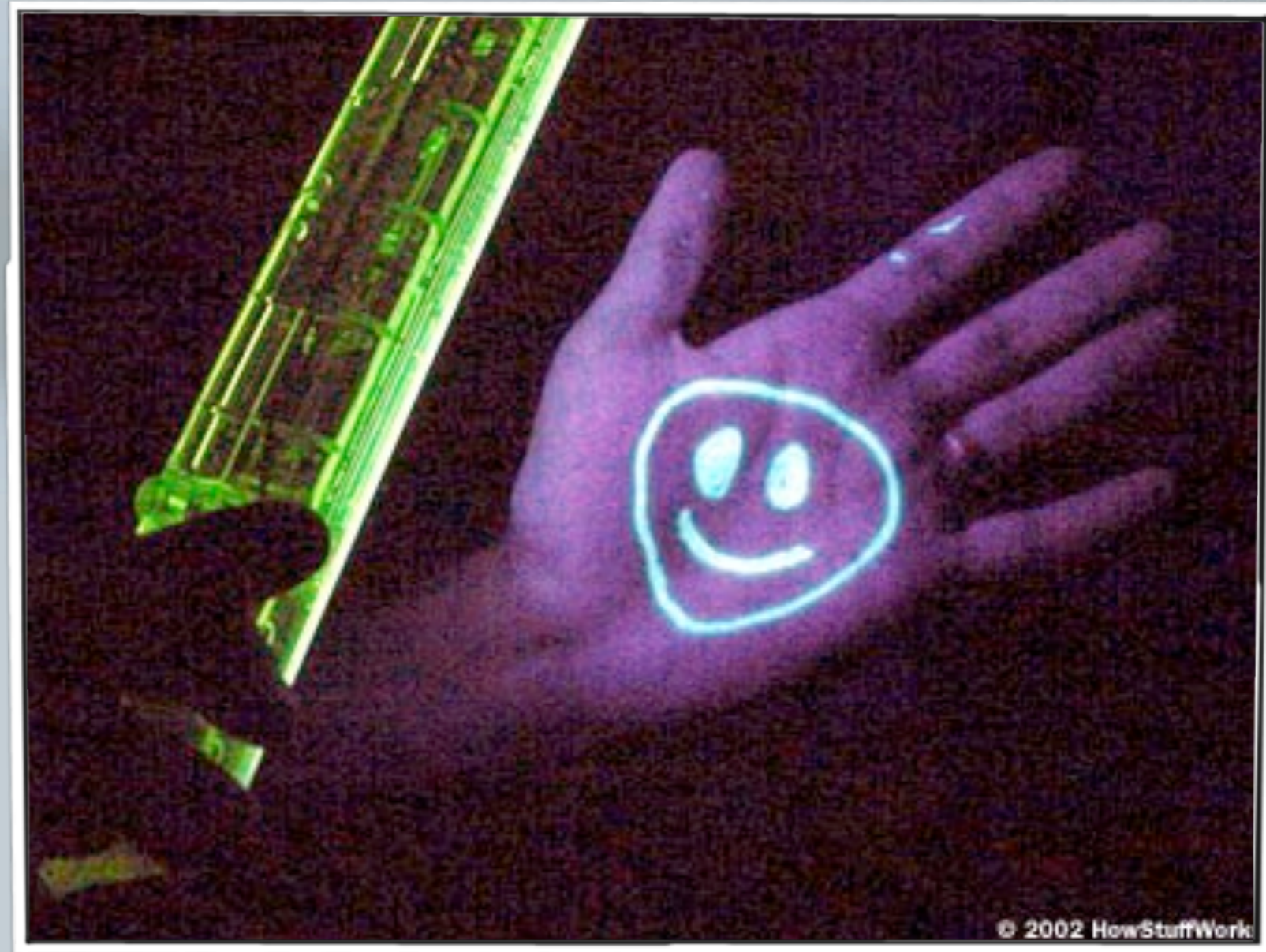


$$\lambda = \frac{h \cdot c}{E}$$

$h$  = Planck constant =  $6.626 \cdot 10^{-34}$  J · s

$c$  = speed of light =  $3 \cdot 10^8$  m · sec<sup>-1</sup>

# HAVE I EVER "SEEN" FLUORESCENCE??



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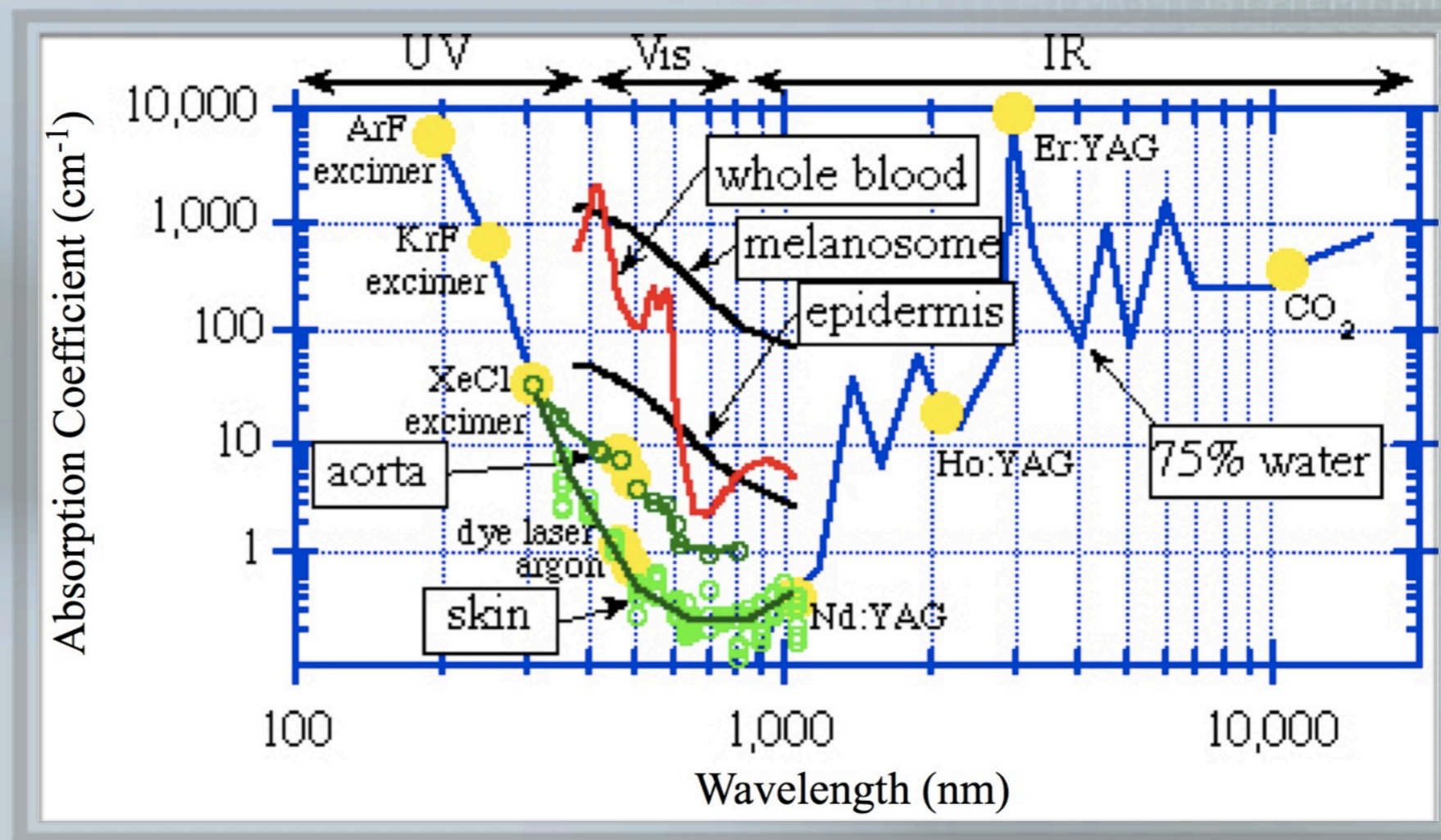
# AUTOFLUORESCENCE OF OUR BODY

- **Autofluorescence:** Emission of fluorescence without the need for exogenous fluorophores.

Chromophore	$\lambda_{exc}$ (nm)	$\lambda_{em}$ (nm)
Tryptophan	275	350
	285	310
Collagen	340	395
	270	395
	285	310
	260	410
Elastin	460	520
	360	410
	425	490
	260	410
NADH	350	460
Endogenous Porphyrins	400	610, 675



# ABSORPTION OF OUR BODY



- Absorption dominated spectral range:  $\lambda < 250$  nm and  $\lambda > 2000$  nm.
- Scattering dominated spectral range:  $600$  nm  $< \lambda < 1200$  nm.  
*Optical window.*
- Comparable scattering and absorption: all the other wavelengths.

# SO...WHY NEAR IR WAVELENGTHS?

- Most of the tissue endogenous fluorophores absorb at the region  $\lambda < 600$  nm.
- The same as blood and most of all other tissue molecules.

**For wavelengths below 600 nm the light presents very small penetration depth.**

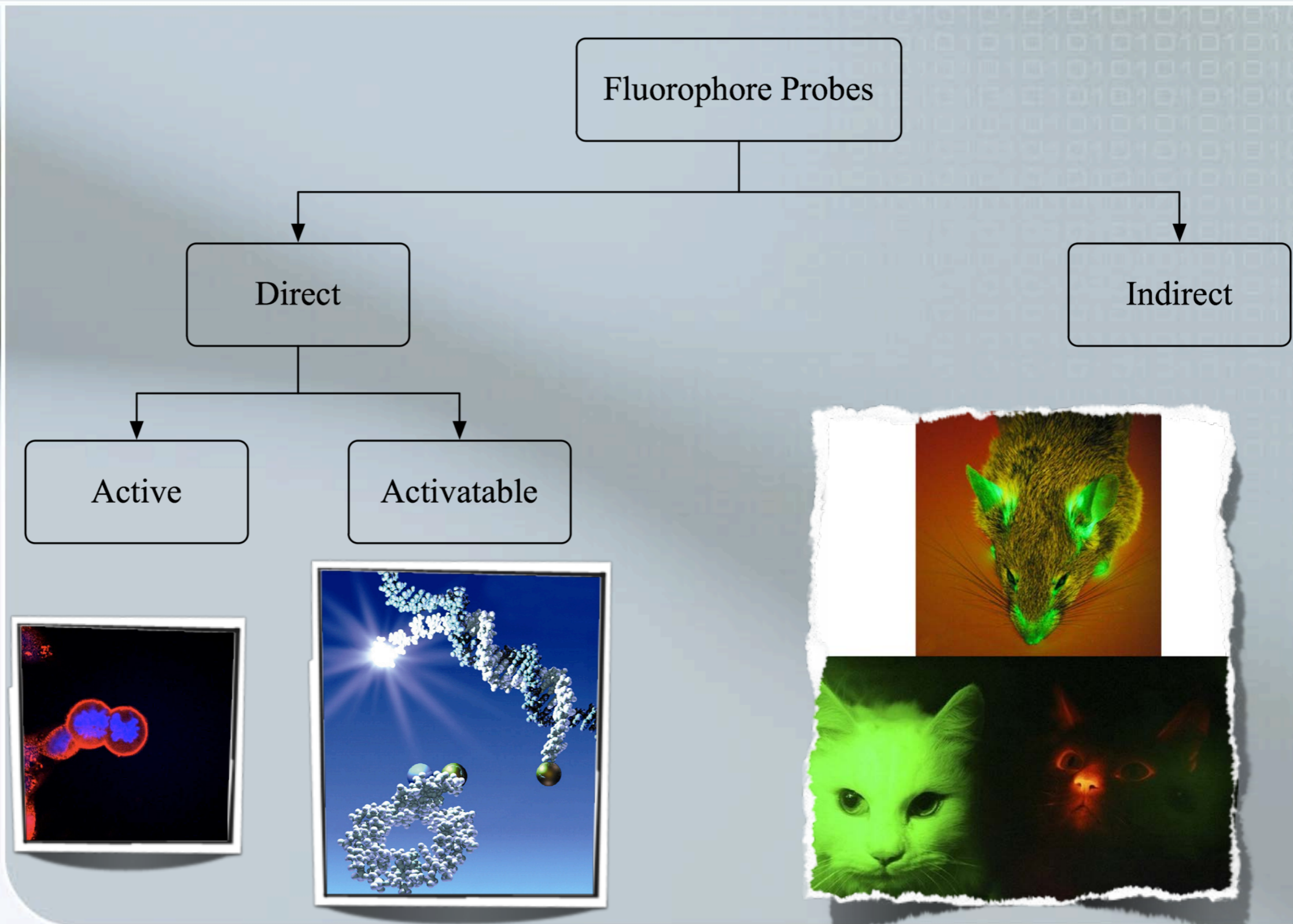
*(Very useful in multi-view machine vision systems with structured light projection)*

- ▶ Within the optical window, tissue is a scattering dominated medium and thus light can propagate a few centimeters before it is absorbed.
- ▶ Autofluorescence does not exist in the optical window and thus the acquired fluorescence is the signal of interest.





# FLUOROPHORES TECHNOLOGY



# OUTLINE

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# FLUORESCENCE MOLECULAR IMAGING

The solution of the reconstruction problem, in the context of fluorescence molecular imaging, corresponds to the estimation of the fluorophores distribution within the investigated medium, when:

- ✦ the amount of guided light and
- ✦ the measured data on the boundary of the object are given.

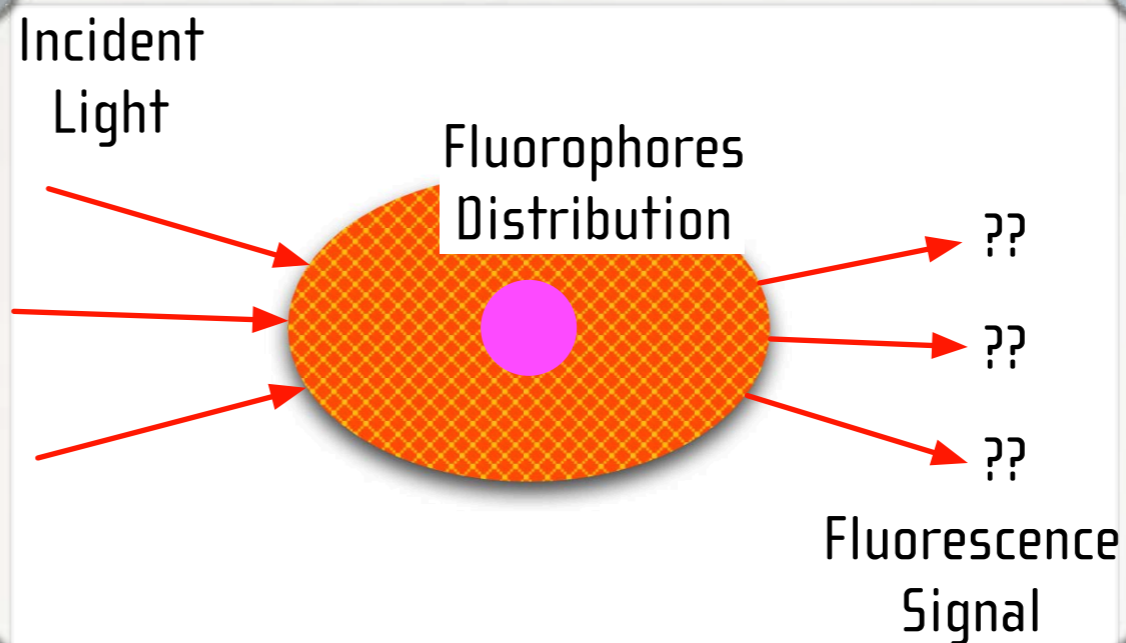
The fluorescence molecular imaging investigated characteristics are:

- \* the absorption coefficient values of the fluorophores,
- \* their three-dimensional distributions and
- \* their fluorescence lifetime.



# THE FORWARD PROBLEM IN FLUORESCENCE MOLECULAR IMAGING

The **forward problem** in fluorescence imaging is to solve the measurable data, which are the intensity values recorded by the detector on the surface of the inspected region, when the fluorescence distribution and the input light sources are given.

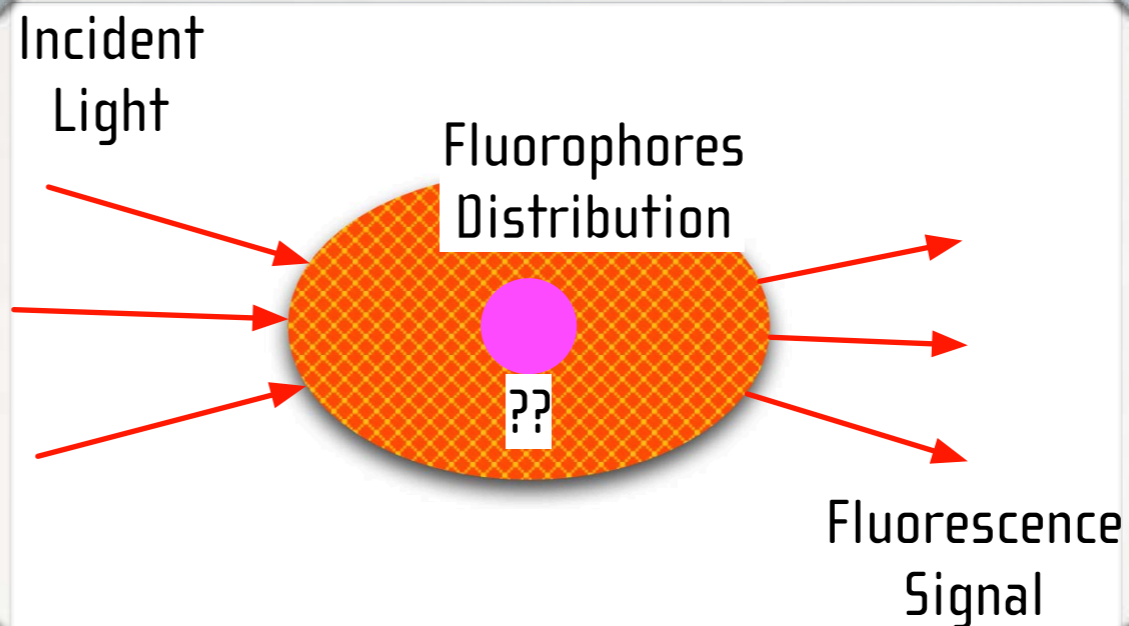


## Forward Solver

- ◆ Monte Carlo (MC) is the gold standard,
- ◆ Diffusion Approximation (DA),
- ◆ Radiative Transfer Equation (RTE) is still under investigation.

# THE INVERSE PROBLEM IN FLUORESCENCE MOLECULAR IMAGING

The **inverse problem** in fluorescence imaging is defined as the 3D reconstruction of a fluorophores distribution embedded into a turbid medium, like tissues, by utilizing the acquired fluorescence signal and the outcomes of the forward problem solution.

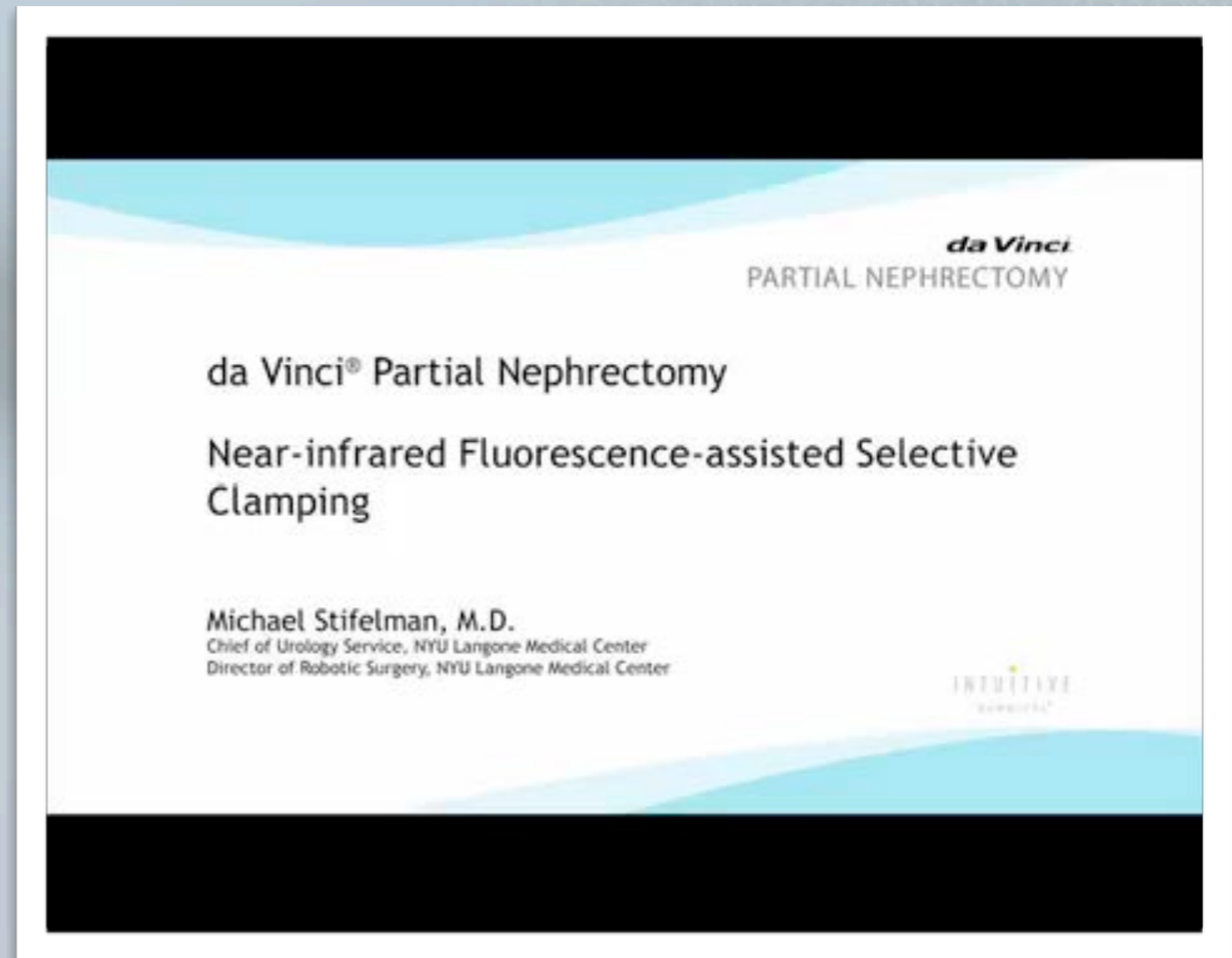


## Inverse Problem

- ◆ Optimization problem is formed,
- ◆ The fluorophores distribution is updated after each iteration,
- ◆ Application of the forward solver to the new distribution,
- ◆ Iterations continue until convergence occurs.

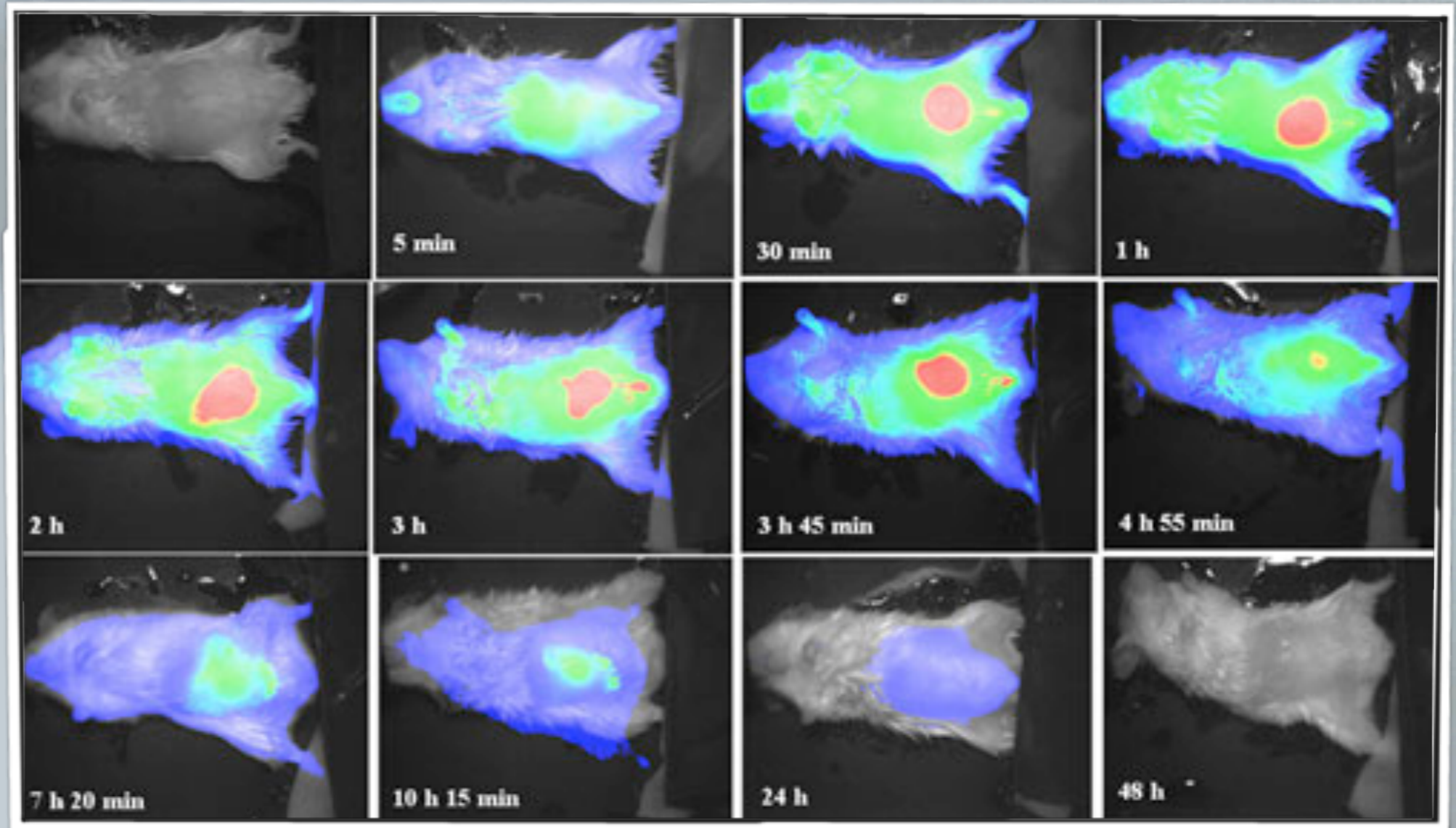
# APPLICATIONS OF THE FLUORESCENCE MOLECULAR IMAGING

Example of current clinical application of the fluorescence molecular imaging



# APPLICATIONS OF THE FLUORESCENCE MOLECULAR IMAGING

Example of fluorescence molecular imaging application in the pharmaceutical field.



Kovar J et al., "A systematic approach to the development of fluorescent contrast agents for optical imaging of mouse cancer models", *Cancer Res.* 69(13):5592-600 (2009).

# APPLICATIONS OF THE FLUORESCENCE MOLECULAR IMAGING

## Possible prospects for clinical application of the fluorescence molecular imaging/tomography

- Breast cancer diagnosis.
- Tumor diagnosis/quantification.
- Photodynamic therapy evaluation.
- Lung inflammation.
- Immunology.
- Cardiology.





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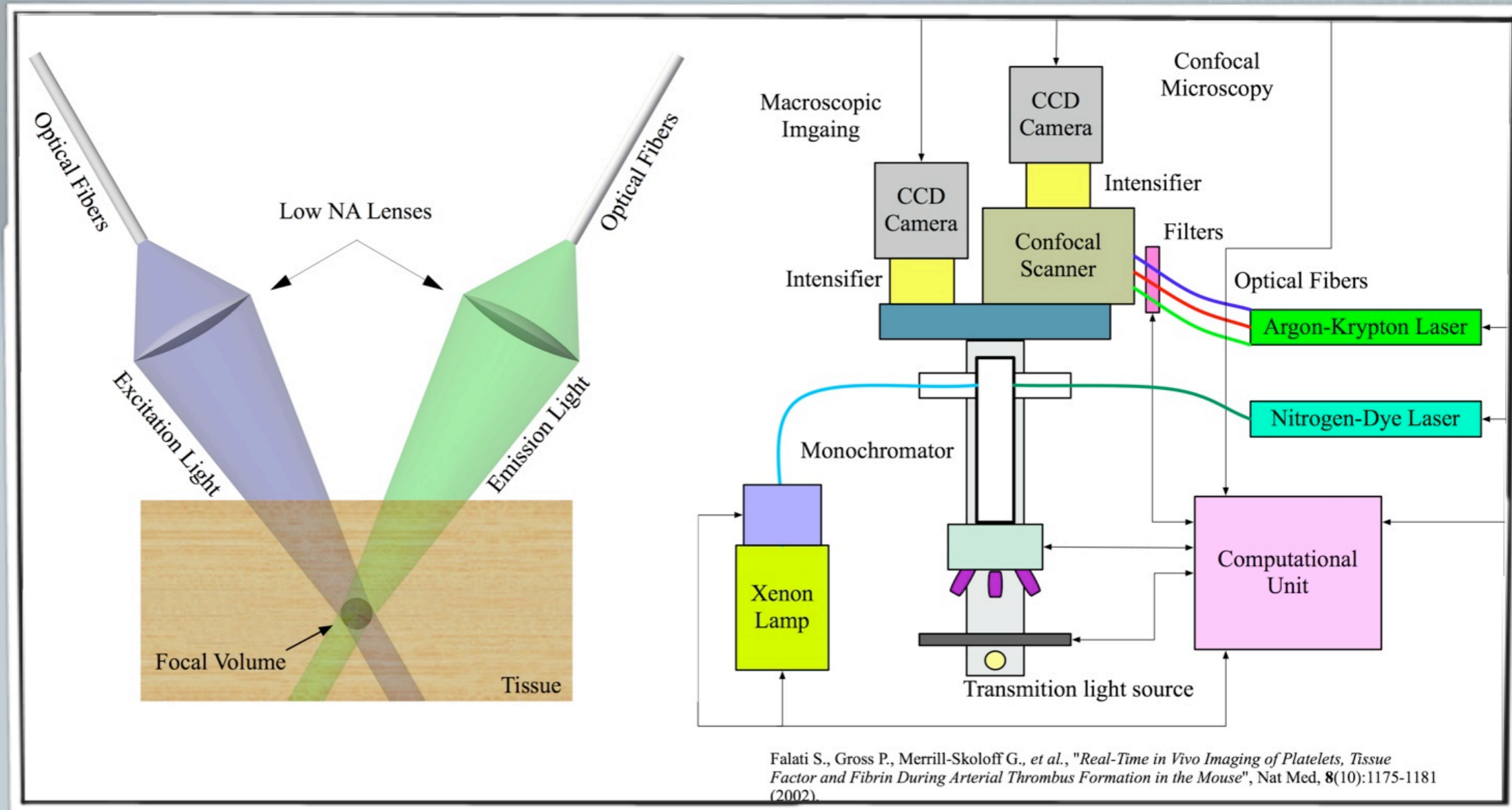
Light propagation models.

Conclusion.



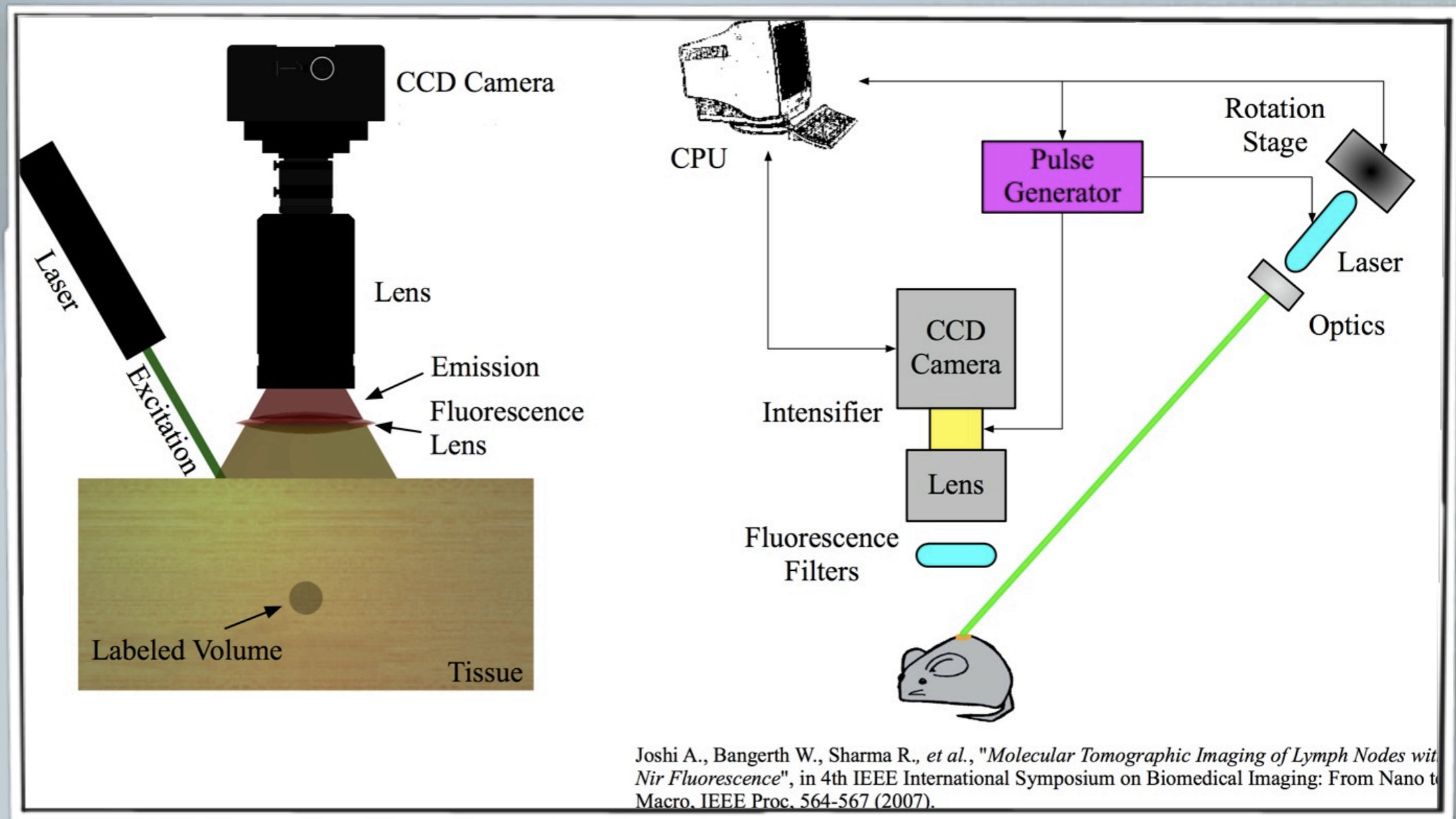
# IMAGING TECHNOLOGIES

## I. *In vivo* fluorescence microscopy



# IMAGING TECHNOLOGIES

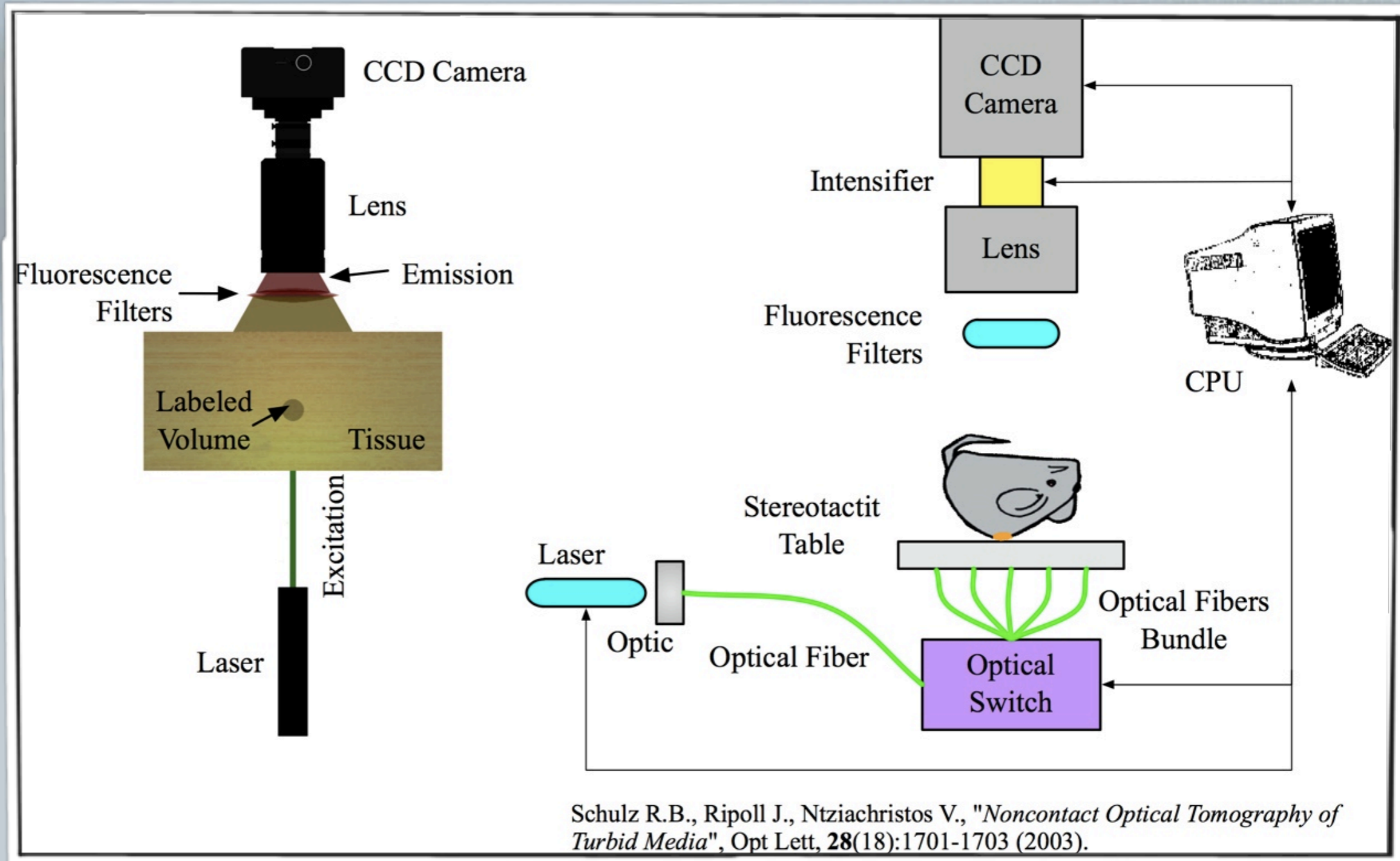
## II. Epi-illumination imaging



Joshi A., Bangerth W., Sharma R., et al., "Molecular Tomographic Imaging of Lymph Nodes with Near Infrared Fluorescence", in 4th IEEE International Symposium on Biomedical Imaging: From Nano to Macro, IEEE Proc. 564-567 (2007).

# IMAGING TECHNOLOGIES

## III. Trans-illumination imaging



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# LIGHT PROPAGATION MODELS - FORWARD PROBLEM SOLUTION

## The Radiative Transfer Equation:

- ◆ is accurate in predicting light propagation,
- ◆ requires huge computing resources and
- ◆ is time consuming.

$$\left. \begin{aligned} \frac{i \cdot \omega}{c} \cdot I_{x/m}(\mathbf{r}, \hat{\mathbf{s}}) + \hat{\mathbf{s}} \cdot \nabla I_{x/m}(\mathbf{r}, \hat{\mathbf{s}}) + [\mu_{\alpha, x/m}(\mathbf{r}) + \mu_{s, x/m}(\mathbf{r})] \cdot I_{x/m}(\mathbf{r}, \hat{\mathbf{s}}) \\ - \mu_{s, x/m}(\mathbf{r}) \cdot \int_{4\pi} p_{x/m}(\hat{\mathbf{s}}, \hat{\mathbf{s}}') \cdot I_{x/m}(\mathbf{r}, \hat{\mathbf{s}}') \cdot d\hat{\mathbf{s}}' = \Lambda_{x/m}(\mathbf{r}, \hat{\mathbf{s}}) \end{aligned} \right|_{\mathbf{r} \in V_{\text{RTE}}}$$

$$I_{x/m}(\mathbf{r}, \hat{\mathbf{s}}) = \begin{cases} 0 & \mathbf{r} \in S_{\text{RTE, out}} \setminus dS_{\text{src}}, \hat{\mathbf{s}} \cdot \hat{\mathbf{n}} < 0 \\ I_{\text{src}}(\mathbf{r}, \hat{\mathbf{s}}) & \mathbf{r} \in dS_{\text{src}}, \hat{\mathbf{s}} \cdot \hat{\mathbf{n}} < 0 \end{cases}$$

## The Diffusion Approximation:

- ◆ is accurate in predicting light propagation when the application is scattered dominated,
- ◆ provides inaccurate predictions close to the light source and
- ◆ provides inaccurate predictions close to the region boundaries.

$$\left. \frac{i \cdot \omega}{c} \cdot U_{x/m}(\mathbf{r}) - \nabla \cdot [D_{x/m}(\mathbf{r}) \cdot \nabla U_{x/m}(\mathbf{r})] + \mu_{\alpha, x/m}(\mathbf{r}) \cdot U_{x/m}(\mathbf{r}) = \Lambda_{0, x/m}(\mathbf{r}) \right|_{\mathbf{r} \in V_{\text{DA}}}$$

$$U_{x/m}(\mathbf{r}) = -2 \cdot A \cdot D_{x/m}(\mathbf{r}) \cdot \frac{\partial}{\partial \hat{\mathbf{n}}} U_{x/m}(\mathbf{r}) \quad \mathbf{r} \in S_{\text{DA, out}}, \hat{\mathbf{s}} \cdot \hat{\mathbf{n}} < 0$$

# LIGHT PROPAGATION MODELS - FORWARD PROBLEM SOLUTION

## Dual Coupled RTE-DA model

RTE

$$\left. \begin{aligned} \frac{i \cdot \omega}{c} \cdot I_{x/m}(\mathbf{r}, \hat{\mathbf{s}}) + \hat{\mathbf{s}} \cdot \nabla I_{x/m}(\mathbf{r}, \hat{\mathbf{s}}) + [\mu_{\alpha, x/m}(\mathbf{r}) + \mu_{s, x/m}(\mathbf{r})] \cdot I_{x/m}(\mathbf{r}, \hat{\mathbf{s}}) \\ - \mu_{s, x/m}(\mathbf{r}) \cdot \int_{4\pi} P_{x/m}(\hat{\mathbf{s}}, \hat{\mathbf{s}}') \cdot I_{x/m}(\mathbf{r}, \hat{\mathbf{s}}') \cdot d\hat{\mathbf{s}}' = \Lambda_{x/m}(\mathbf{r}, \hat{\mathbf{s}}) \end{aligned} \right|_{\mathbf{r} \in V_{\text{RTE}}}$$

Vacuum boundary condition

$$I_{x/m}(\mathbf{r}, \hat{\mathbf{s}}) = \begin{cases} 0 & \mathbf{r} \in S_{\text{RTE, out}} \setminus dS_{\text{src}}, \hat{\mathbf{s}} \cdot \hat{\mathbf{n}} < 0 \\ I_{\text{src}}(\mathbf{r}, \hat{\mathbf{s}}) & \mathbf{r} \in dS_{\text{src}}, \hat{\mathbf{s}} \cdot \hat{\mathbf{n}} < 0 \end{cases}$$

Interface boundary condition

$$I_{x/m}(\mathbf{r}, \hat{\mathbf{s}}) = U_{x/m}(\mathbf{r}) - \frac{3}{4 \cdot \pi} \cdot [D_{x/m}(\mathbf{r}) \cdot \nabla U_{x/m}(\mathbf{r})] \cdot \hat{\mathbf{s}} \Big|_{\mathbf{r} \in S_{\text{interface}}}$$

DA

$$\left. \frac{i \cdot \omega}{c} \cdot U_{x/m}(\mathbf{r}) - \nabla [D_{x/m}(\mathbf{r}) \cdot \nabla U_{x/m}(\mathbf{r})] + \mu_{\alpha, x/m}(\mathbf{r}) \cdot U_{x/m}(\mathbf{r}) = \Lambda_{0, x/m}(\mathbf{r}) \right|_{\mathbf{r} \in V_{\text{DA}}}$$

Robin type boundary condition

$$U_{x/m}(\mathbf{r}) = -2 \cdot A \cdot D_{x/m}(\mathbf{r}) \cdot \frac{\partial}{\partial \hat{\mathbf{n}}} U_{x/m}(\mathbf{r}) \quad \mathbf{r} \in S_{\text{DA, out}}, \hat{\mathbf{s}} \cdot \hat{\mathbf{n}} < 0$$

Interface boundary condition

$$U_{x/m}(\mathbf{r}) = \frac{1}{4 \cdot \pi} \cdot \int_{4\pi} I_{x/m}(\mathbf{r}, \hat{\mathbf{s}}) \cdot d\hat{\mathbf{s}} \quad \mathbf{r} \in S_{\text{interface}}$$

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$$\Lambda_x(\mathbf{r}, \hat{\mathbf{s}}) = \Lambda_{0, x}(\mathbf{r}) = 0$$

$$\Lambda_m(\mathbf{r}, \hat{\mathbf{s}}) = \Lambda_{0, m}(\mathbf{r}) = \frac{\eta \cdot \mu_{\alpha, x}^{\text{fluo}}(\mathbf{r})}{1 + i \cdot \omega \cdot \tau(\mathbf{r})} \cdot U_x(\mathbf{r})$$

DA

$$\left. \frac{i \cdot \omega}{c} \cdot U_{x/m}(\mathbf{r}) - \nabla [D_{x/m}(\mathbf{r}) \cdot \nabla U_{x/m}(\mathbf{r})] + \mu_{\alpha, x/m}(\mathbf{r}) \cdot U_{x/m}(\mathbf{r}) \in \Lambda_{0, x/m}(\mathbf{r}) \right|_{\mathbf{r} \in V_{\text{DA}}}$$

Robin type boundary condition

$$U_{x/m}(\mathbf{r}) = -2 \cdot A \cdot D_{x/m}(\mathbf{r}) \cdot \frac{\partial}{\partial \hat{\mathbf{n}}} U_{x/m}(\mathbf{r}) \quad \mathbf{r} \in S_{\text{DA, out}}, \hat{\mathbf{s}} \cdot \hat{\mathbf{n}} < 0$$

Interface boundary condition

$$U_{x/m}(\mathbf{r}) = \frac{1}{4 \cdot \pi} \cdot \int_{4\pi} I_{x/m}(\mathbf{r}, \hat{\mathbf{s}}) \cdot d\hat{\mathbf{s}} \quad \mathbf{r} \in S_{\text{interface}}$$



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Interface boundary condition

$$\begin{aligned} \mu_{s, x/m}(\mathbf{r}) &= \mu_{s, x/m}^{\text{tis}}(\mathbf{r}) \\ \mu_{\alpha, x/m}(\mathbf{r}) &= \mu_{\alpha, x/m}^{\text{tis}}(\mathbf{r}) + \mu_{\alpha, x/m}^{\text{fluo}}(\mathbf{r}) \end{aligned}$$

$$\begin{aligned} \Lambda_x(\mathbf{r}, \hat{\mathbf{s}}) &= \Lambda_{0, x}(\mathbf{r}) = 0 \\ \Lambda_m(\mathbf{r}, \hat{\mathbf{s}}) &= \Lambda_{0, m}(\mathbf{r}) = \frac{\eta \cdot \mu_{\alpha, x}^{\text{fluo}}(\mathbf{r})}{1 + i \cdot \omega \cdot \tau(\mathbf{r})} \cdot U_x(\mathbf{r}) \end{aligned}$$

DA

$$\left. \begin{aligned} \frac{i \cdot \omega}{c} \cdot U_{x/m}(\mathbf{r}) - \nabla \cdot [D_{x/m}(\mathbf{r}) \cdot \nabla U_{x/m}(\mathbf{r})] + \mu_{\alpha, x/m}(\mathbf{r}) \cdot U_{x/m}(\mathbf{r}) \in \Lambda_{0, x/m}(\mathbf{r}) \end{aligned} \right|_{\mathbf{r} \in V_{\text{DA}}}$$

Robin type boundary condition

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Interface boundary condition

$$U_{x/m}(\mathbf{r}) = \frac{1}{4 \cdot \pi} \cdot \int_{4\pi} I_{x/m}(\mathbf{r}, \hat{\mathbf{s}}) \cdot d\hat{\mathbf{s}} \quad \mathbf{r} \in S_{\text{interface}}$$

# THE DUAL COUPLED RTE-DA MODEL

## Numerical solution of the forward problem

The finite elements method is the one adopted as the numerical solution approximation of the forward problem in fluorescence molecular imaging.



# THE DUAL COUPLED RTE-DA MODEL

## Numerical solution of the forward problem

- ▶ Variational formulation of the dual coupled RTE-DA model.
  - ▶ RTE is multiplied with the test function  $\psi(\mathbf{r}, \hat{\mathbf{s}})$  and integrated over the domain  $V_{\text{RTE}}$  and for all the angular directions.
  - ▶ DA is multiplied with the test function  $y(\mathbf{r})$  and integrated over the domain  $V_{\text{DA}}$ .
- ▶ Finite elements approximation of the dual coupled RTE-DA model.
  - ▶ The solutions of the variational formulation are approximated in piece-wise linear functions per element (standard Galerkin technique).

$$I_{x/m}(\mathbf{r}, \hat{\mathbf{s}}) \approx I_{x/m}^h(\mathbf{r}, \hat{\mathbf{s}}) = \sum_{i=1}^{N_n} \sum_{l=1}^{N_a} \alpha_{il,x/m} \cdot \psi_i(\mathbf{r}) \cdot \psi_l(\hat{\mathbf{s}})$$

$$U_{x/m}(\mathbf{r}) \approx U_{x/m}^h(\mathbf{r}) = \sum_{k=1}^N a_{k,x/m} \cdot y_k(\mathbf{r})$$

# THE DUAL COUPLED RTE-DA MODEL

## Numerical solution of the forward problem

- ▶ Application of the streamline diffusion modification, sdm, for minimization of the “photon rays”.

$$\psi(\mathbf{r}, \hat{\mathbf{s}}) := \psi(\mathbf{r}, \hat{\mathbf{s}}) + \delta(\mathbf{r}) \cdot \hat{\mathbf{s}} \cdot \nabla \psi(\mathbf{r}, \hat{\mathbf{s}})$$

- ▶ Transformation of the resulted linear algebraic system into its matrix formalism.

$$\begin{bmatrix} \mathbf{A}_{\text{RTE},x/m} & \mathbf{B}_{\text{RTE},x/m} \\ \mathbf{B}_{\text{DA},x/m} & \mathbf{A}_{\text{DA},x/m} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{\alpha}_{x/m} \\ \mathbf{a}_{x/m} \end{bmatrix} = \begin{bmatrix} \mathbf{C}_{\text{RTE},x/m} \\ \mathbf{C}_{\text{DA},x/m} \end{bmatrix}$$

$$\mathbf{A}_{\text{RTE},x/m}(\mathbf{h}_1, \mathbf{h}_2) = \mathbf{A}_0^{x/m}(\mathbf{h}_1, \mathbf{h}_2) + \mathbf{A}_1^{x/m}(\mathbf{h}_1, \mathbf{h}_2) + \mathbf{A}_2^{x/m}(\mathbf{h}_1, \mathbf{h}_2) + \mathbf{A}_3^{x/m}(\mathbf{h}_1, \mathbf{h}_2) + \mathbf{A}_4^{x/m}(\mathbf{h}_1, \mathbf{h}_2)$$

$$\mathbf{A}_{\text{DA},x/m}(\mathbf{p}, \mathbf{k}) = \mathbf{K}\mathbf{e}_{x/m}(\mathbf{p}, \mathbf{k}) + \mathbf{M}\mathbf{e}_{x/m}(\mathbf{p}, \mathbf{k}) + \mathbf{P}\mathbf{e}_{x/m}(\mathbf{p}, \mathbf{k})$$

# THE DUAL COUPLED RTE-DA MODEL

## Numerical solution of the forward problem

- ▶ Application of the streamline diffusion modification, sdm, for minimization of the “photon rays”.

- ▶ Translating the matrix

$$\mathbf{A}_0^{x/m}(h_1, h_2) = \frac{i \cdot \omega}{c} \cdot \int_{V_{\text{RTE}}} \psi_i(\mathbf{r}) \cdot \psi_j(\mathbf{r}) \cdot d\mathbf{r} \cdot \int_{4\pi} \psi_l(\hat{\mathbf{s}}) \cdot \psi_q(\hat{\mathbf{s}}) \cdot d\hat{\mathbf{s}} + \frac{i \cdot \omega}{c} \cdot \int_{V_{\text{RTE}}} \int_{4\pi} \delta_{x/m}(\mathbf{r}) \cdot \hat{\mathbf{s}} \cdot \nabla \psi_j(\mathbf{r}) \cdot \psi_q(\hat{\mathbf{s}}) \cdot \psi_l(\hat{\mathbf{s}}) \cdot d\hat{\mathbf{s}} \cdot \psi_i(\mathbf{r}) \cdot d\mathbf{r}$$

$$\left[ \mathbf{B}_{\text{DA},x/m} \quad \mathbf{A}_{\text{DA},x/m} \right] \left[ \mathbf{a}_{x/m} \right] \left[ \mathbf{C}_{\text{DA},x/m} \right]$$

$$\mathbf{A}_{\text{RTE},x/m}(h_1, h_2) = \mathbf{A}_0^{x/m}(h_1, h_2) + \mathbf{A}_1^{x/m}(h_1, h_2) + \mathbf{A}_2^{x/m}(h_1, h_2) + \mathbf{A}_3^{x/m}(h_1, h_2) + \mathbf{A}_4^{x/m}(h_1, h_2)$$

$$\mathbf{A}_{\text{DA},x/m}(p, k) = \mathbf{K} \mathbf{e}_{x/m}(p, k) + \mathbf{M} \mathbf{e}_{x/m}(p, k) + \mathbf{P} \mathbf{e}_{x/m}(p, k)$$

# THE DUAL COUPLED RTE-DA MODEL

## Numerical solution of the forward problem

- ▶ Application of the streamline diffusion modification, sdm, for minimization of the “photon rays”.

$$\psi(\mathbf{r}, \hat{\mathbf{s}}) := \psi(\mathbf{r}, \hat{\mathbf{s}}) + \delta(\mathbf{r}) \cdot \hat{\mathbf{s}} \cdot \nabla \psi(\mathbf{r}, \hat{\mathbf{s}})$$

- ▶ Transformation of the resulted linear algebraic system into its matrix formalism.

$$\begin{bmatrix} \mathbf{A}_{\text{RTE},x/m} & \mathbf{B}_{\text{RTE},x/m} \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha}_{x/m} \end{bmatrix} = \begin{bmatrix} \mathbf{C}_{\text{RTE},x/m} \end{bmatrix}$$

$$\mathbf{K}e_{x/m}(p, k) = \frac{i \cdot \omega}{c} \cdot \int_{V_{\text{DA}}} y_k(\mathbf{r}) \cdot y_p(\mathbf{r}) \cdot d\mathbf{r} + \int_{V_{\text{DA}}} \mu_{\alpha, x/m}(\mathbf{r}) \cdot y_k(\mathbf{r}) \cdot y_p(\mathbf{r}) \cdot d\mathbf{r}$$

$$\mathbf{A}_{\text{RTE},x/m}(h_1, h_2) = \mathbf{A}_0^{x/m}(h_1, h_2) + \mathbf{A}_1^{x/m}(h_1, h_2) + \mathbf{A}_2^{x/m}(h_1, h_2) + \mathbf{A}_3^{x/m}(h_1, h_2) + \mathbf{A}_4^{x/m}(h_1, h_2)$$

$$\mathbf{A}_{\text{DA},x/m}(p, k) = \mathbf{K}e_{x/m}(p, k) + \mathbf{M}e_{x/m}(p, k) + \mathbf{P}e_{x/m}(p, k)$$

# THE DUAL COUPLED RTE-DA MODEL

## Numerical solution of the forward problem

- ▶ Excitation solution and application of the outcomes for the solution of the emission.
  - ▶ The linear systems can be solved through application of the BiCGStab method.
- ▶ Spatial and angular integrals were confronted separately.
  - ▶ Assembly of the finite elements matrices with application of the Kronecker product.

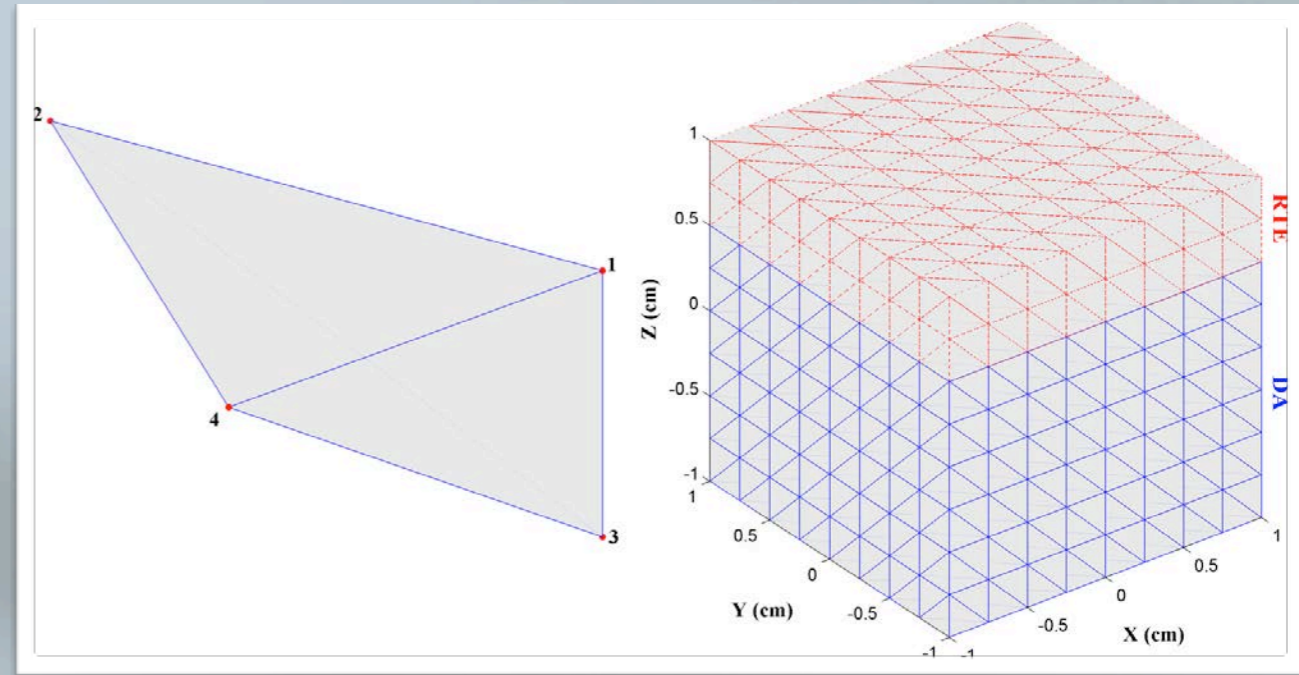
$$\mathbf{C} = \mathbf{A} \otimes \mathbf{B} = \begin{bmatrix} a_{11} \cdot \mathbf{B} & \cdots & a_{n1} \cdot \mathbf{B} \\ \vdots & \ddots & \vdots \\ a_{1m} \cdot \mathbf{B} & \cdots & a_{nm} \cdot \mathbf{B} \end{bmatrix}$$

Gorpas D., Yova D., Politopoulos K., "A Three-dimensional Finite Elements Approach for the Coupled Radiative Transfer Equation and Diffusion Approximation Modeling in Fluorescence Imaging", J. Quant. Spectrosc. Radiat. Transfer, 111(4): 569-584 (2010).

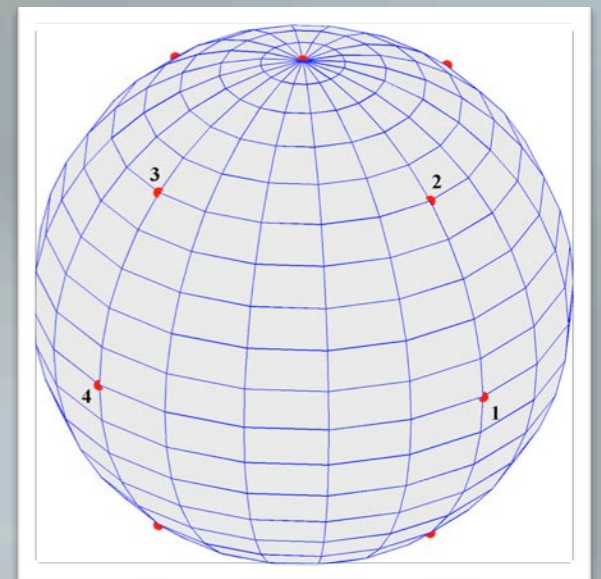
# REGION DISCRETIZATION

## Discretization of the geometrical model

- Spatial discretization with application of the Delaunay triangulation method.



- Angular discretization with application of the Azimuthal technique.





# SUPER-ELLIPSOIDAL MODELS

## Optical properties distribution

- The volume optical properties distribution was implemented through application of the super-ellipsoidal models.

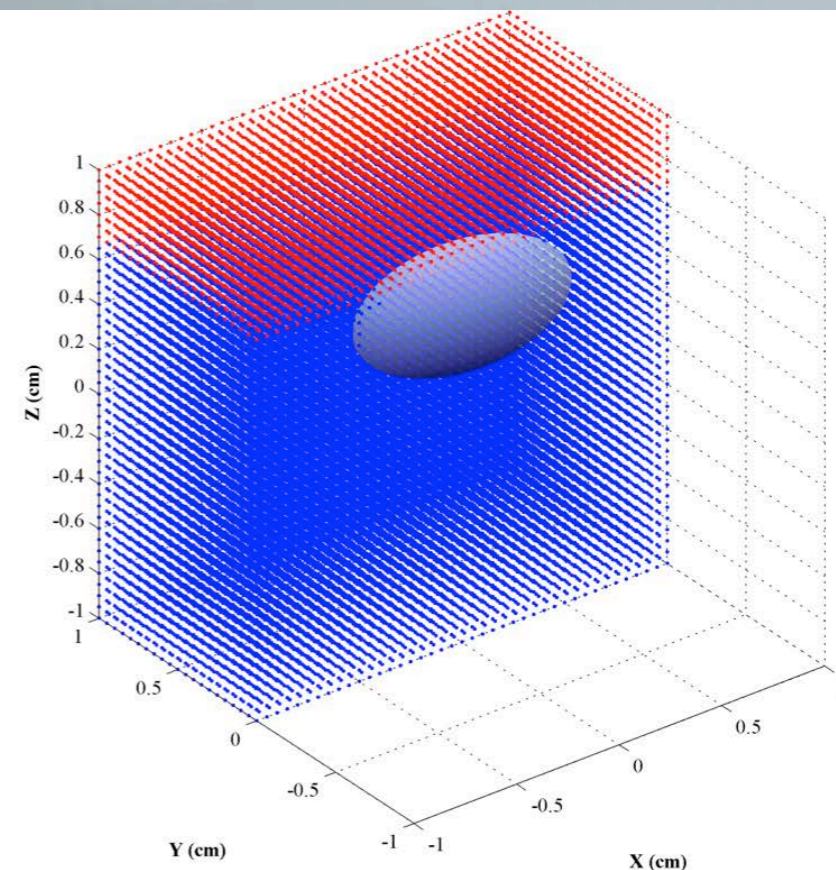
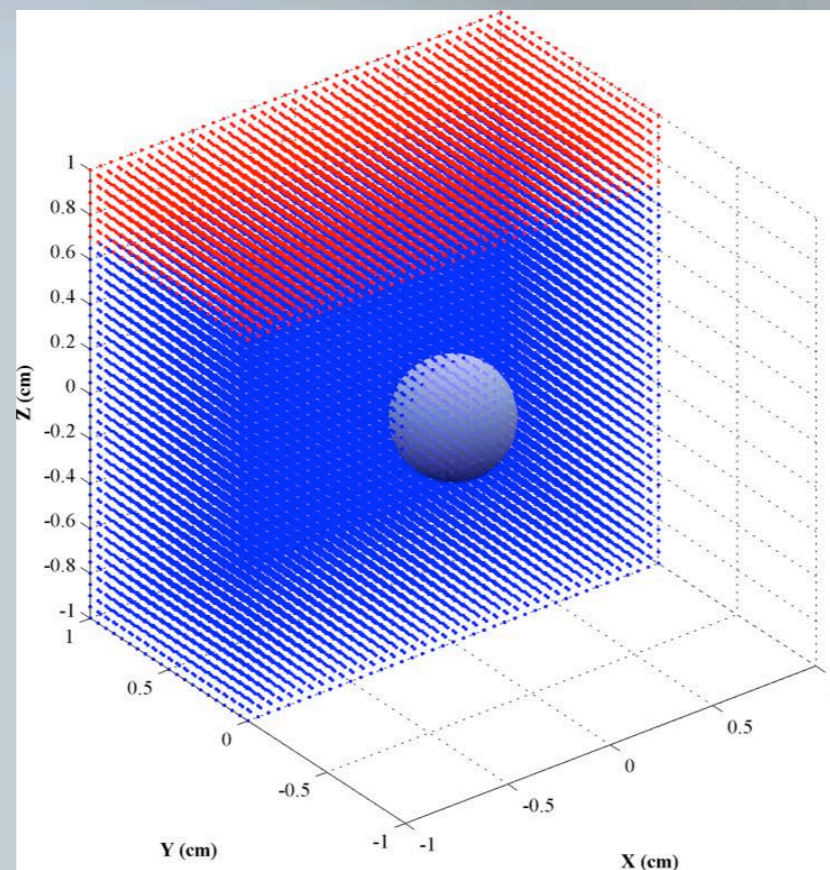
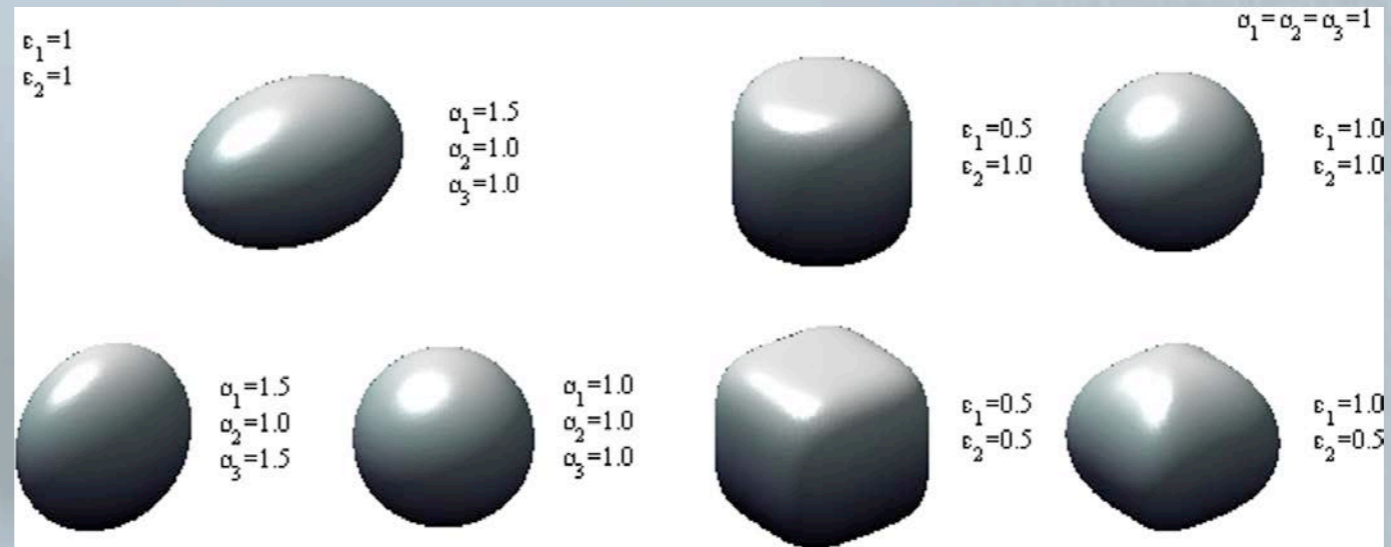
$$f(x, y, z) = \left[ \left( \frac{x}{a_1} \right)^{2/\varepsilon_2} + \left( \frac{y}{a_2} \right)^{2/\varepsilon_2} \right]^{\varepsilon_2/\varepsilon_1} + \left( \frac{z}{a_3} \right)^{2/\varepsilon_1}$$

- Description of the position of a three-dimensional point, related to the super-ellipsoid surface.
  - $f(x, y, z) = 1$  when the point is located on the super-ellipsoid surface,
  - $f(x, y, z) < 1$  when the point is located inside the super-ellipsoid model and
  - $f(x, y, z) > 1$  when the point is located outside the super-ellipsoid model.
- Spatial distribution of the absorption coefficient.

$$\mu_{\alpha, x/m}(\mathbf{r}) = \mu_{\alpha, x/m}^{\text{tis}} + q(\mathbf{r}) \cdot \mu_{\alpha, x/m}^{\text{fluo}} \quad q(\mathbf{r}) = \begin{cases} 1, f(x, y, z) \leq 1 \\ 0, f(x, y, z) > 1 \end{cases}$$

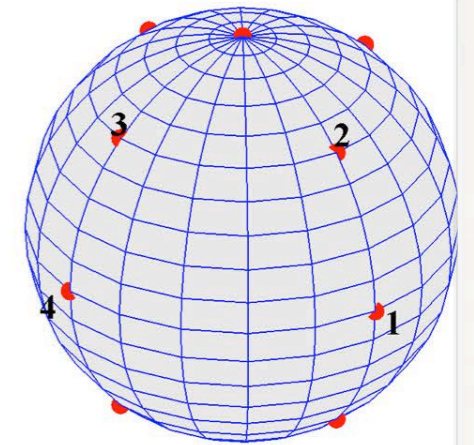
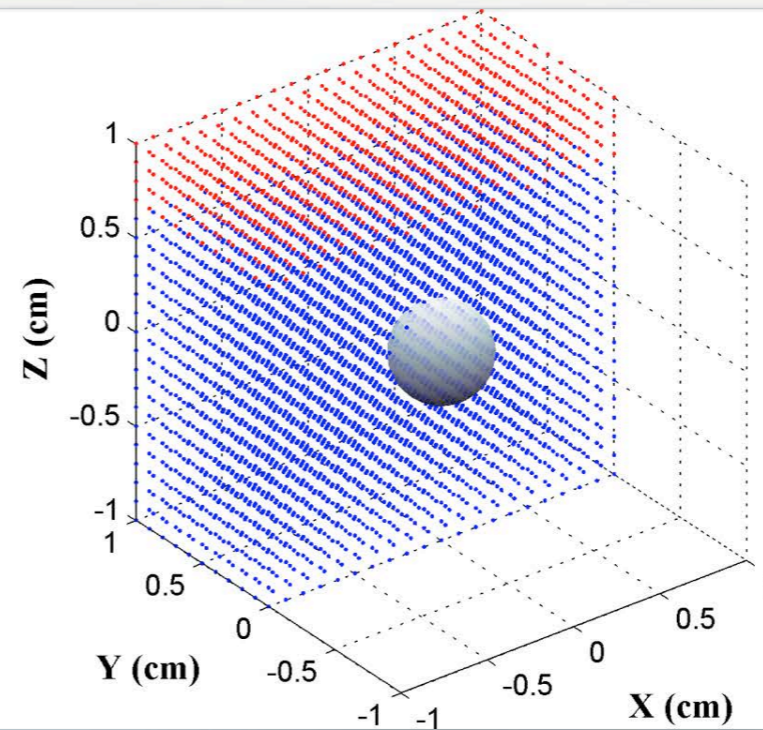
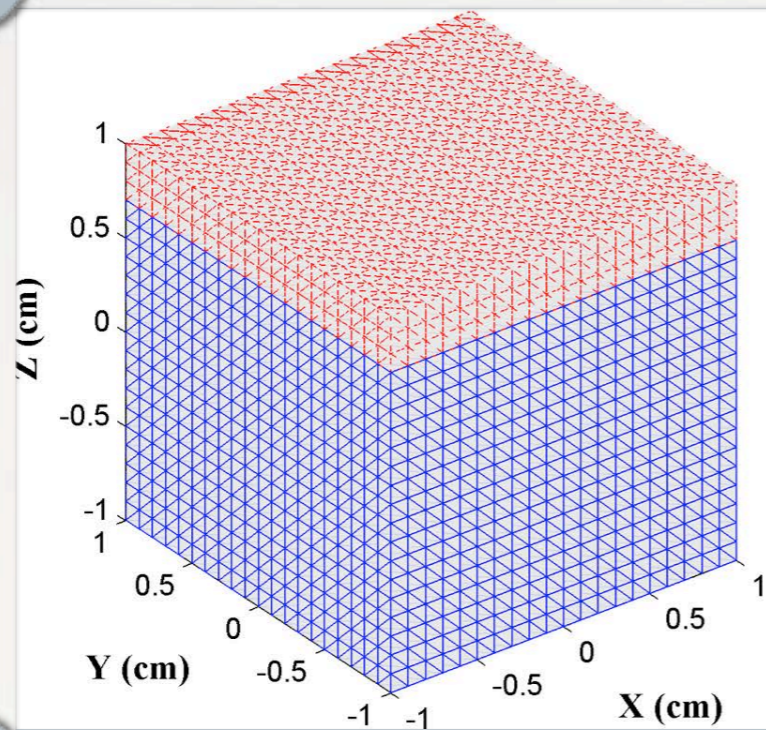
# SUPER-ELLIPSOIDAL MODELS

## Optical properties distribution



# EVALUATION IN FREQUENCY DOMAIN

## Results in the frequency domain




# EVALUATION IN FREQUENCY DOMAIN

## Results in the frequency domain

Spatial discretization:  $\ell = 0.1$  cm  
 Angular discretization:  $N_\theta \times N_\varphi = 4 \times 4$   
 Interface location:  $z = 0.7$  cm

Super-ellipsoidal model with radius  $r = 0.25$  cm, located at the centre of a cubic region with dimensions  $2 \times 2 \times 2$  cm<sup>3</sup>.

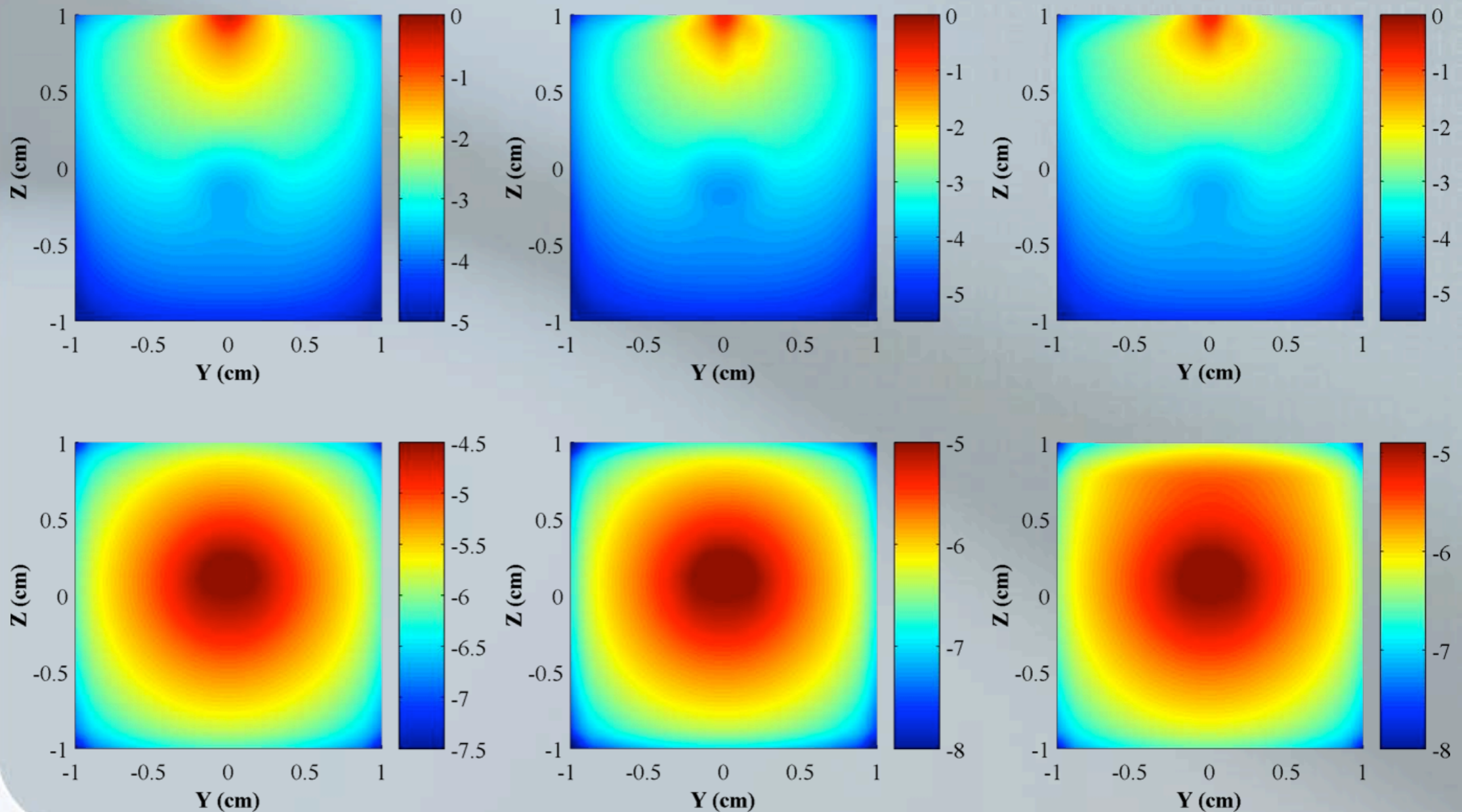


	DA	RTE	RTE-DA
Tetrahedral Elements	48000	48000	7200-RTE 40800-DA
Nodal Points	9261	9261	1764-RTE 7938-DA
Angular Elements	-	16	16-RTE
Angular Nodes	-	20	20-RTE
Assembly Matrices Dimensions	[9261 × 9261]	[185220 × 185220]	[43218 × 43218]
Required Times (sec)	7.69	7.95	8.22

# EVALUATION IN FREQUENCY DOMAIN

## Results in the frequency domain

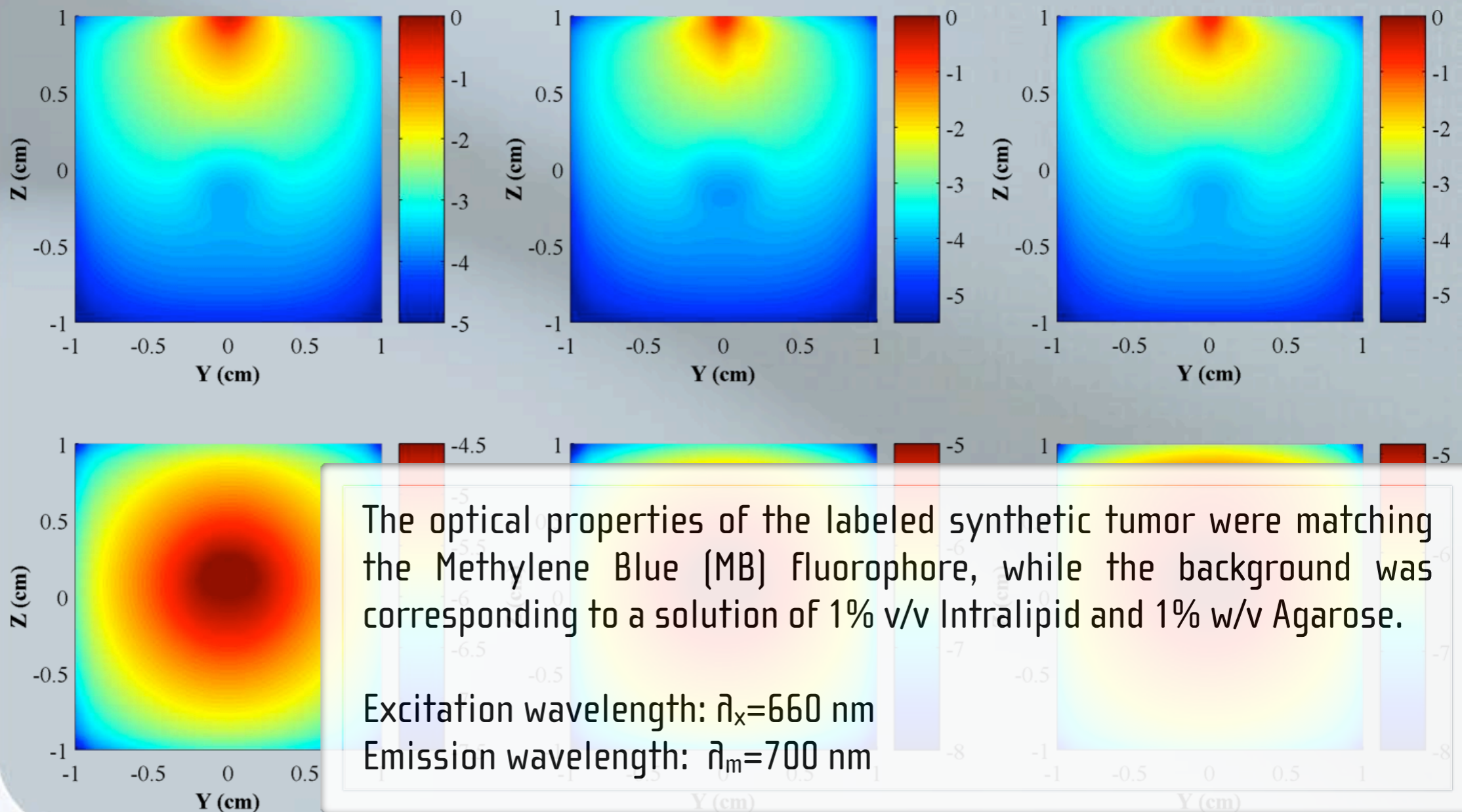
- Logarithm of the photon density modulation amplitude.



# EVALUATION IN FREQUENCY DOMAIN

## Results in the frequency domain

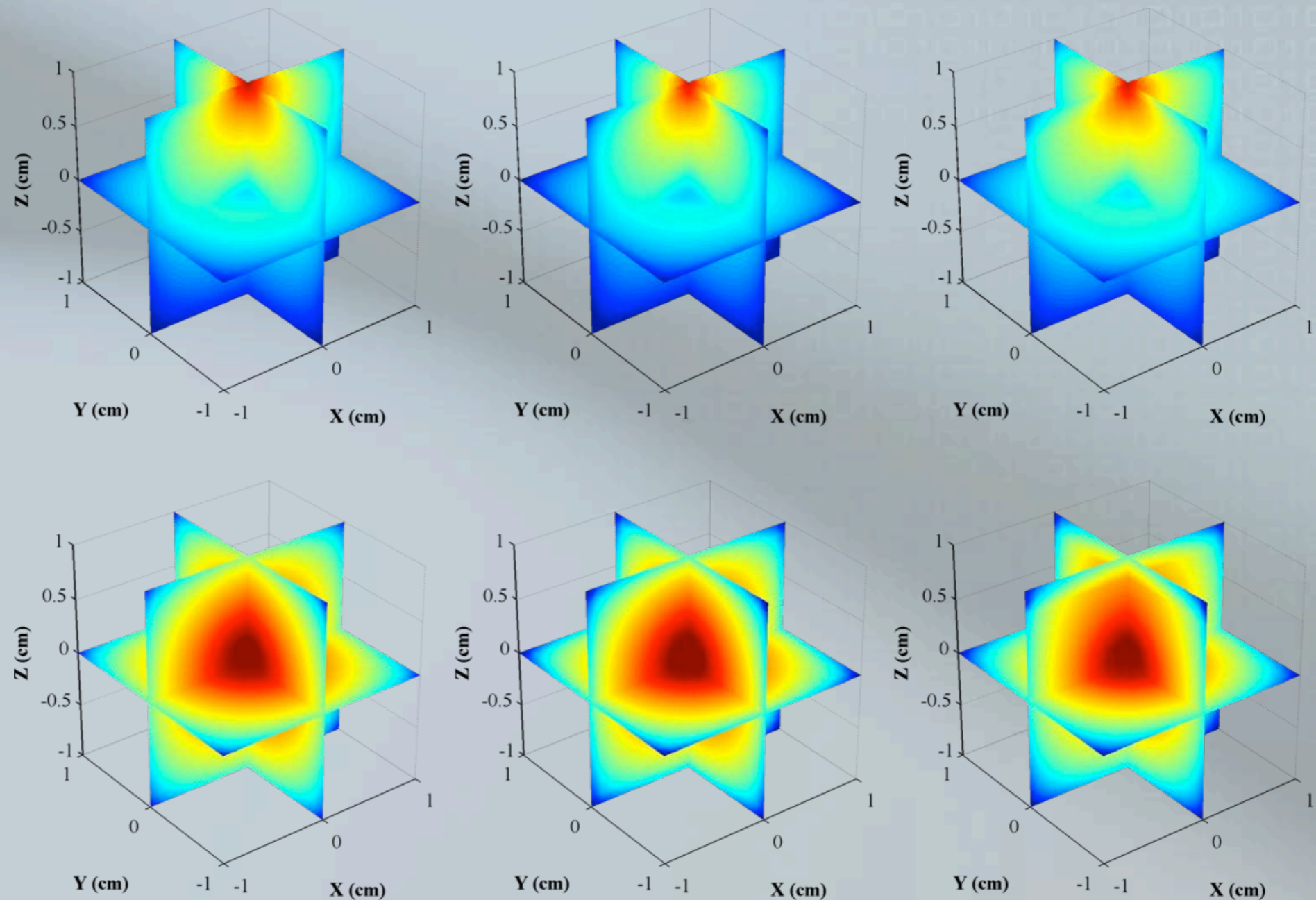
- Logarithm of the photon density modulation amplitude.



# EVALUATION IN FREQUENCY DOMAIN

## Results in the frequency domain

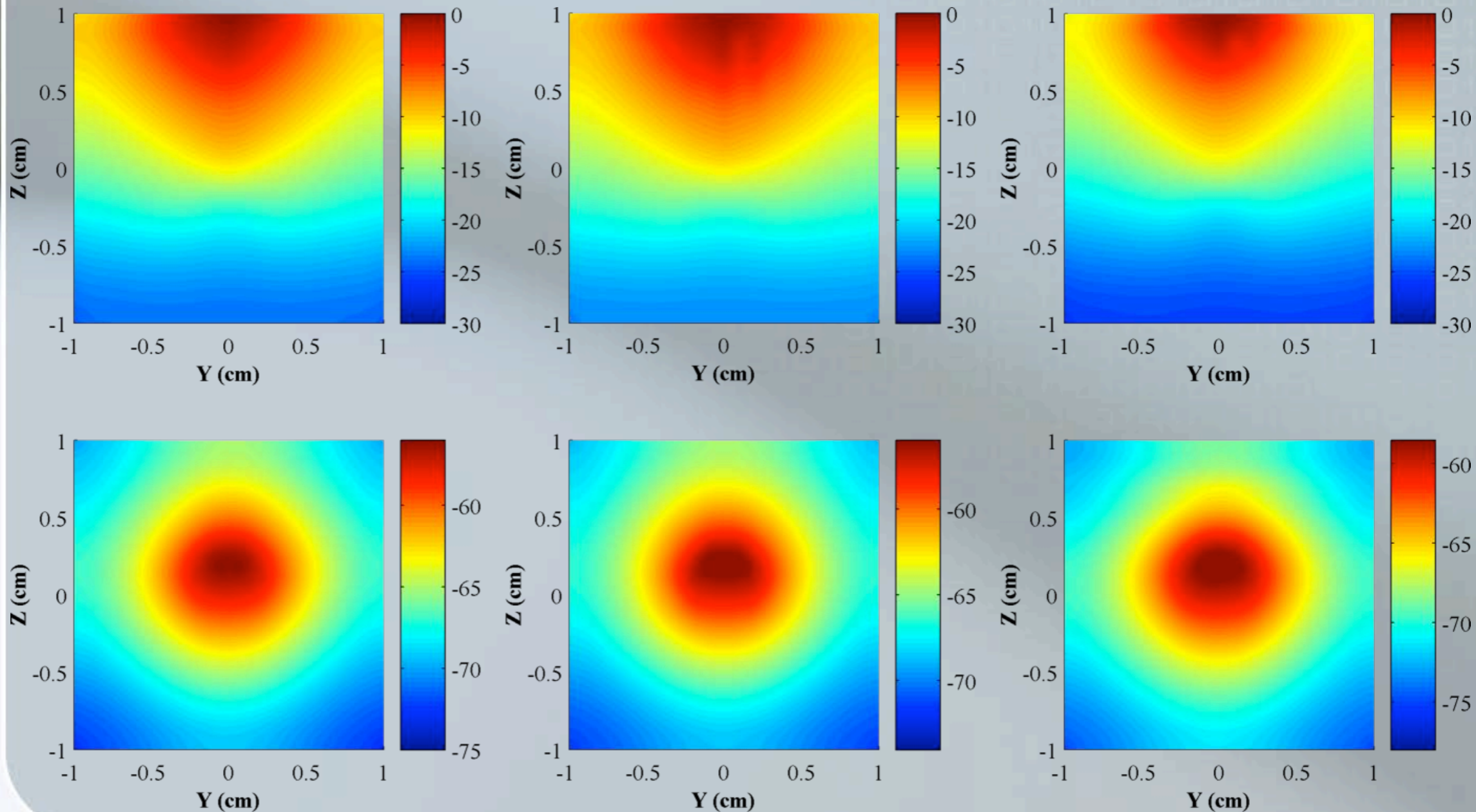
- Logarithm of the photon density modulation amplitude.



# EVALUATION IN FREQUENCY DOMAIN

## Results in the frequency domain

- Phase shift.

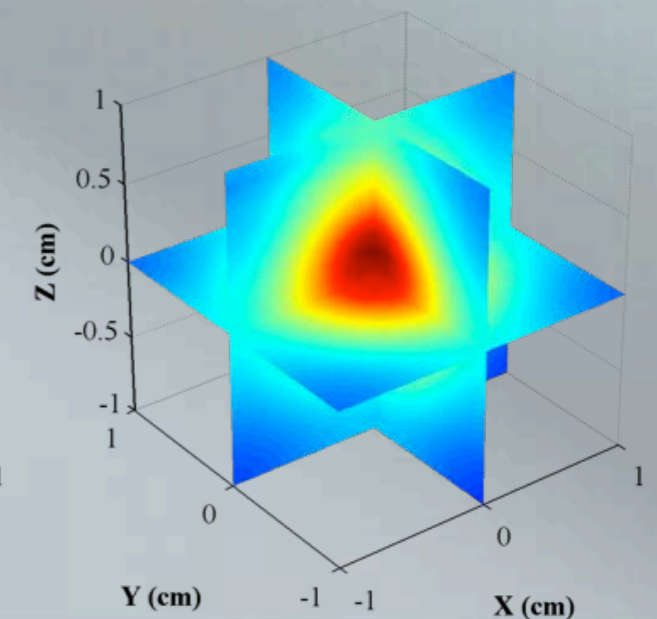
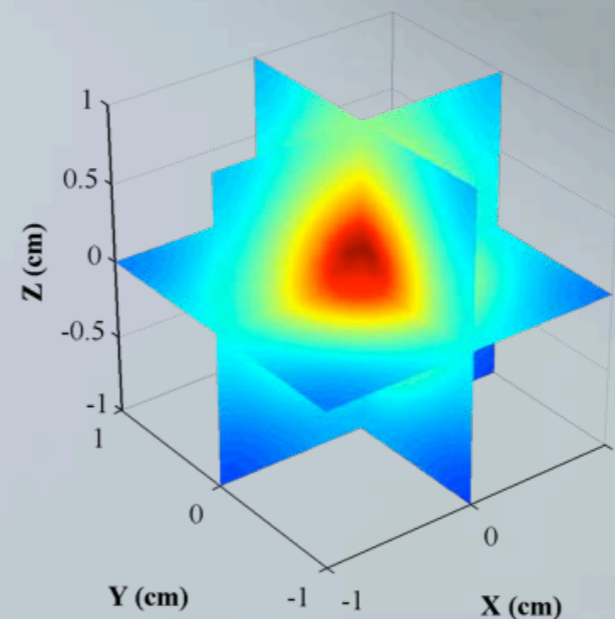
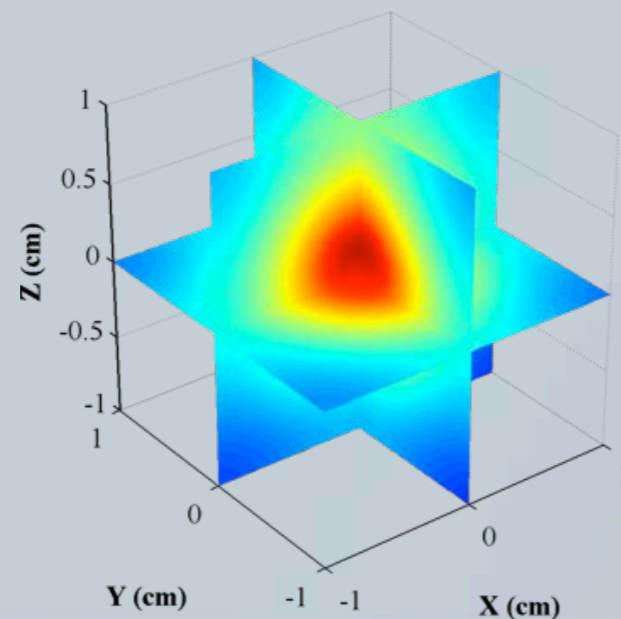
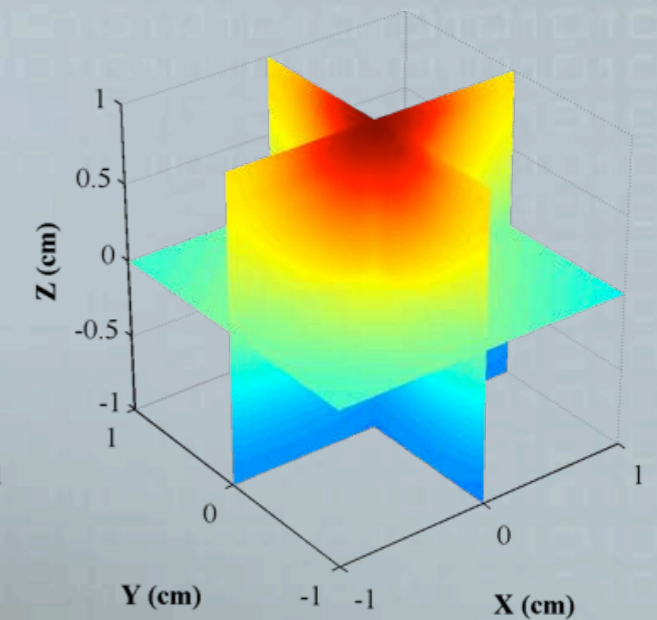
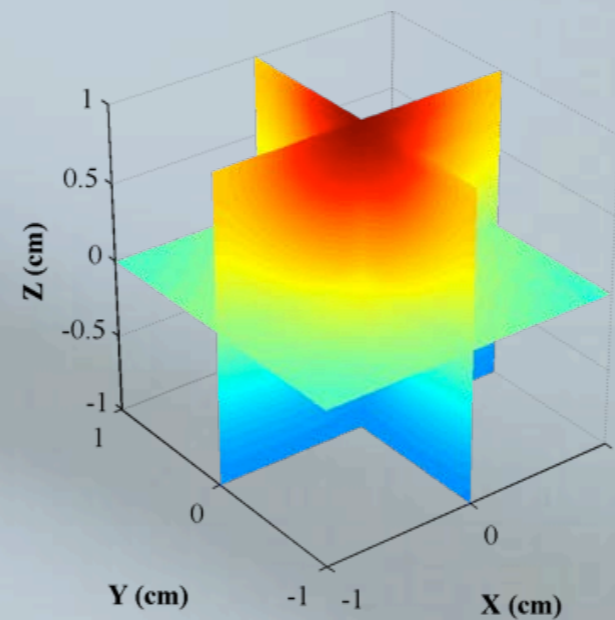
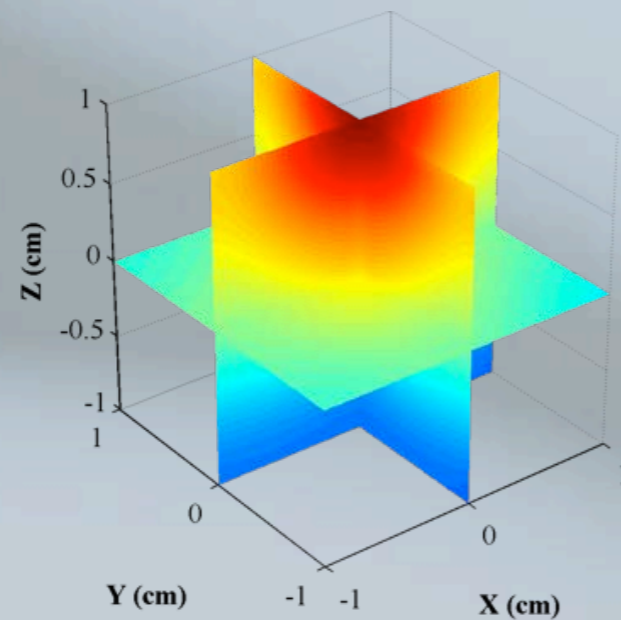




# EVALUATION IN FREQUENCY DOMAIN

## Results in the frequency domain

- Phase shift.



# EVALUATION IN FREQUENCY DOMAIN

## Evaluation of the results in the frequency domain

- Accuracy of the method.



# EVALUATION IN FREQUENCY DOMAIN

## Evaluation of the results in the frequency domain

- Accuracy of the method.
  - Estimated in regards to the RTE.



# EVALUATION IN FREQUENCY DOMAIN

## Evaluation of the results in the frequency domain

- Accuracy of the method.
  - Estimated in regards to the RTE.
  - Estimated through the absolute value of the relative error between the results of the dual coupled RTE-DA model and the RTE based model.

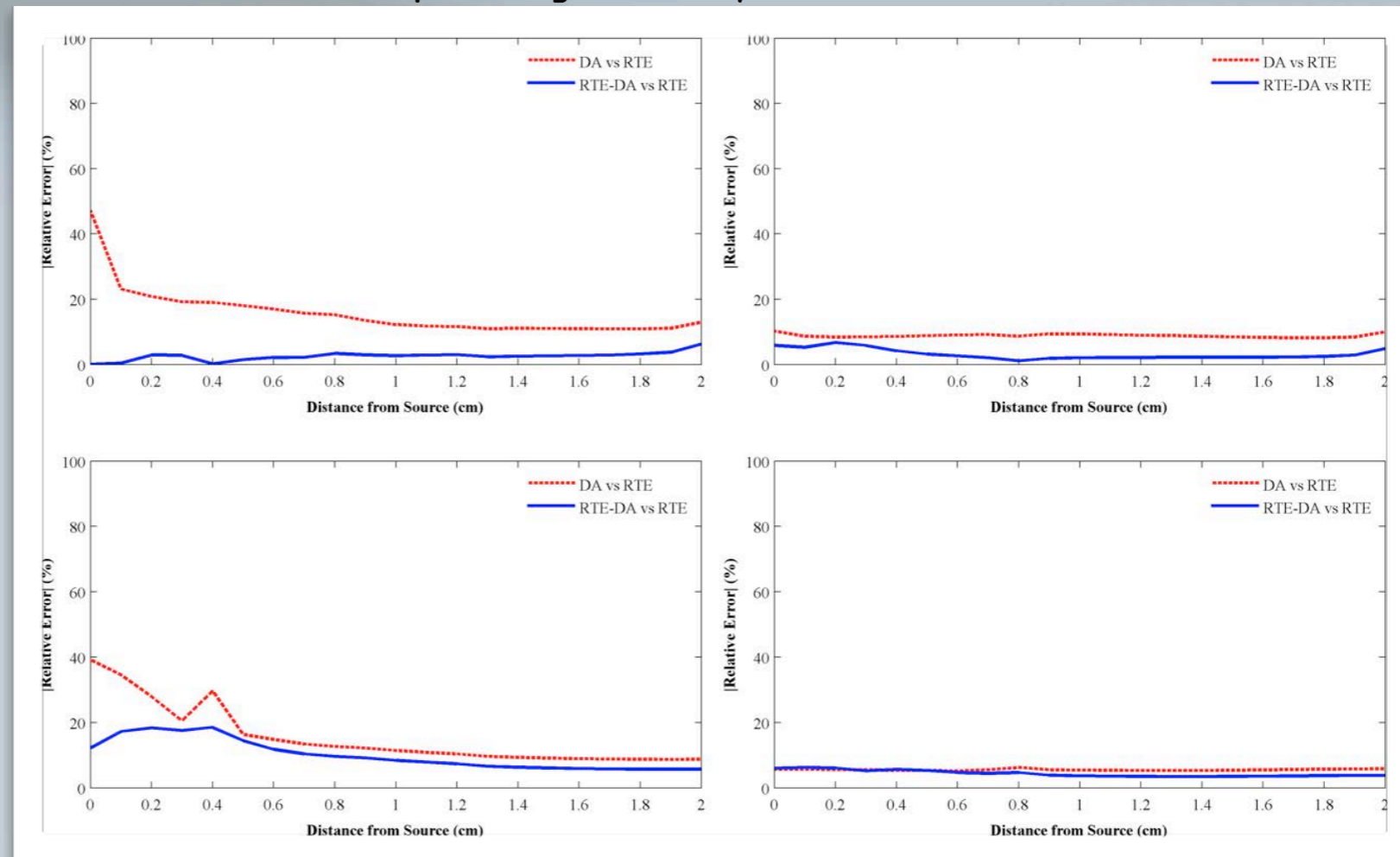
$$\text{ARE}_{A/\theta,DA/RTE-DA} = \left| \frac{M_{A/\theta,RTE} - M_{A/\theta,DA/RTE-DA}}{M_{A/\theta,RTE}} \right| \times 100\%$$



# EVALUATION IN FREQUENCY DOMAIN

## Evaluation of the results in the frequency domain

- Accuracy of the method.
  - Estimated in regards to the RTE.
  - Estimated through the absolute value of the relative error between the results of the dual coupled RTE-DA model and the RTE based model.
  - Compared with the corresponding accuracy of the DA based model.

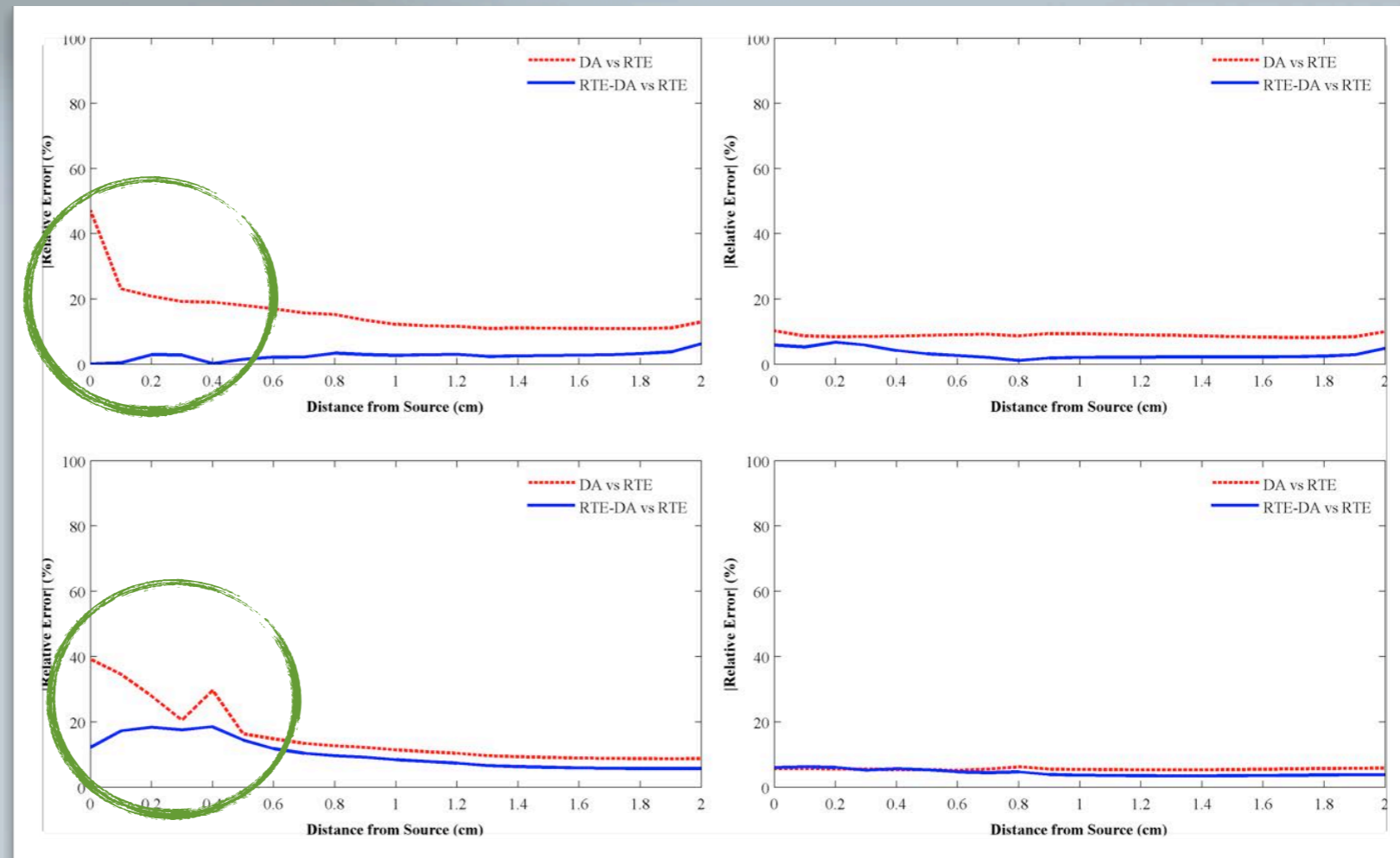


# EVALUATION IN FREQUENCY DOMAIN

## Evaluation of the results in the frequency domain

- Accuracy of the method.

	RTE-DA/RTE		DA/RTE	
	Excitation	Emission	Excitation	Emission
Photon density	~98%	~96%	~85%	~90%
Phase shift	~90%	~96%	~85%	~94%



# EVALUATION IN FREQUENCY DOMAIN

## Evaluation of the results in the frequency domain

- The computational time, the number of iterations and the size of the formulated matrices.

	DA	RTE	RTE-DA
$t_{\text{total}}$ (sec)	38.15	1060.19	480.74
$n_x$	73	101	127
$n_m$	76	165	197
$N_{\text{nz}}$	128 581	48 346 456	8 415 584
$N_{\text{total}}$	85 766 121	34 306 448 400	1 867 795 524

# EVALUATION IN FREQUENCY DOMAIN

## Evaluation of the results in the frequency domain

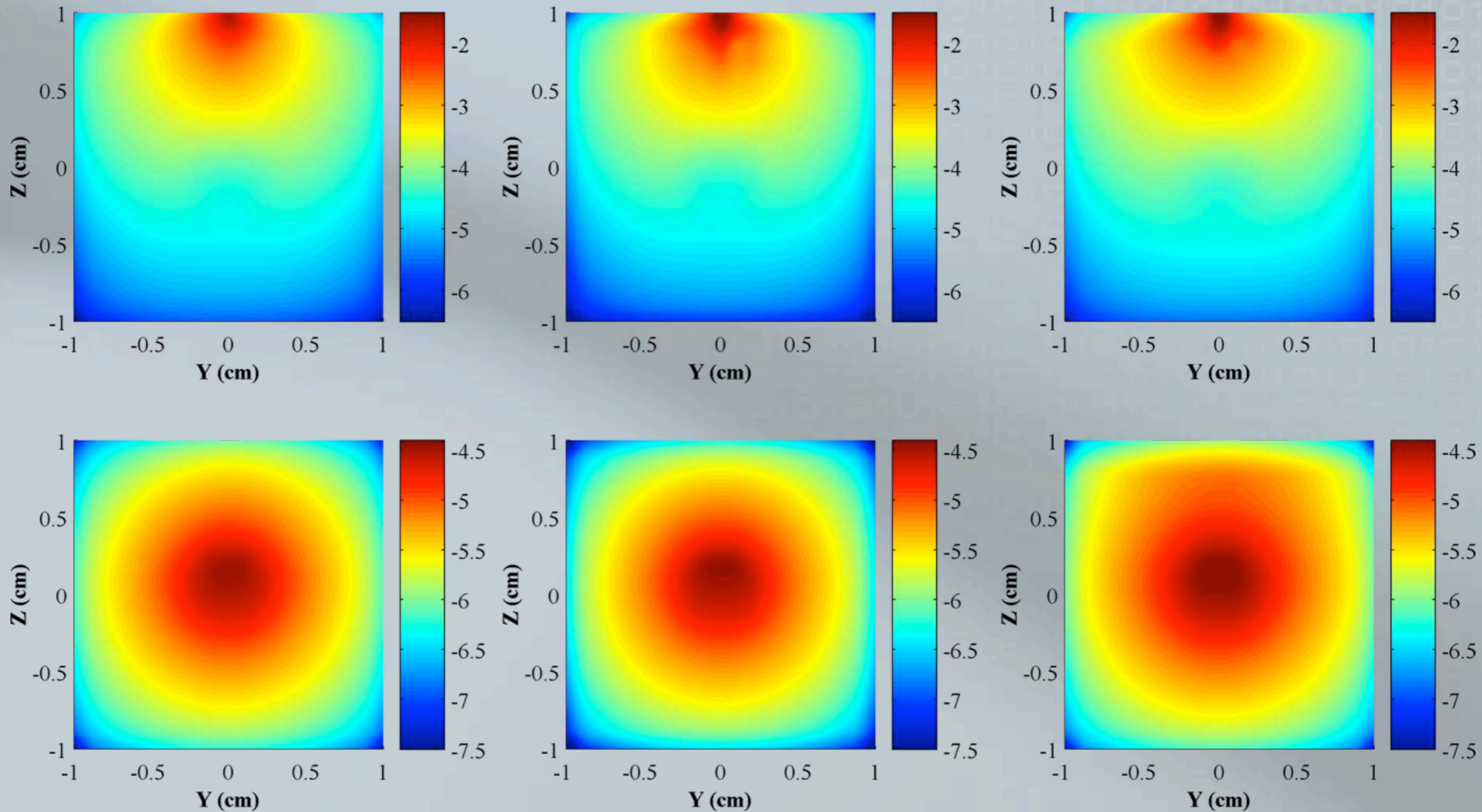
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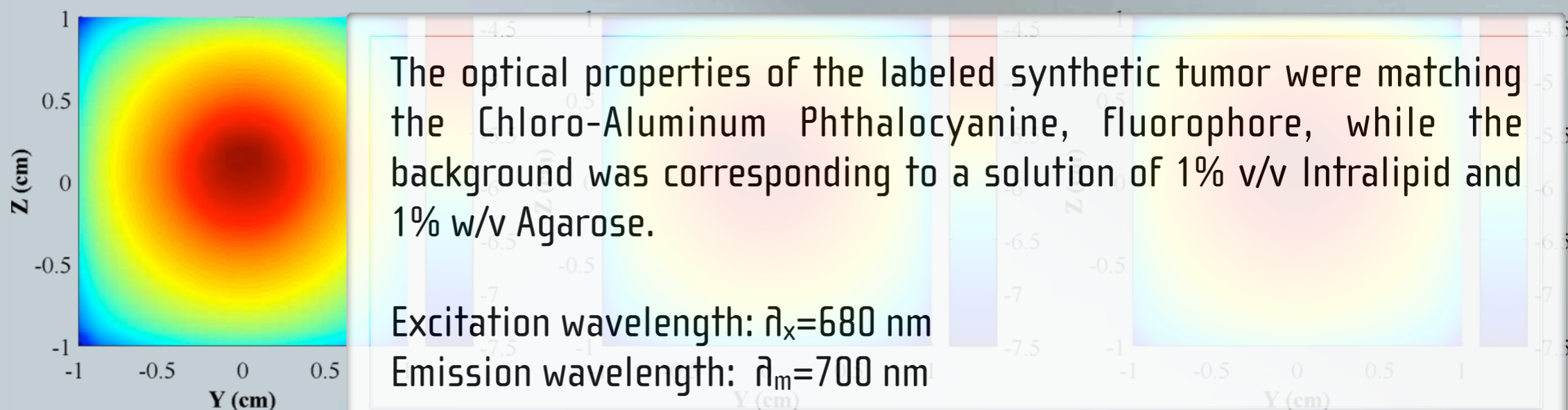
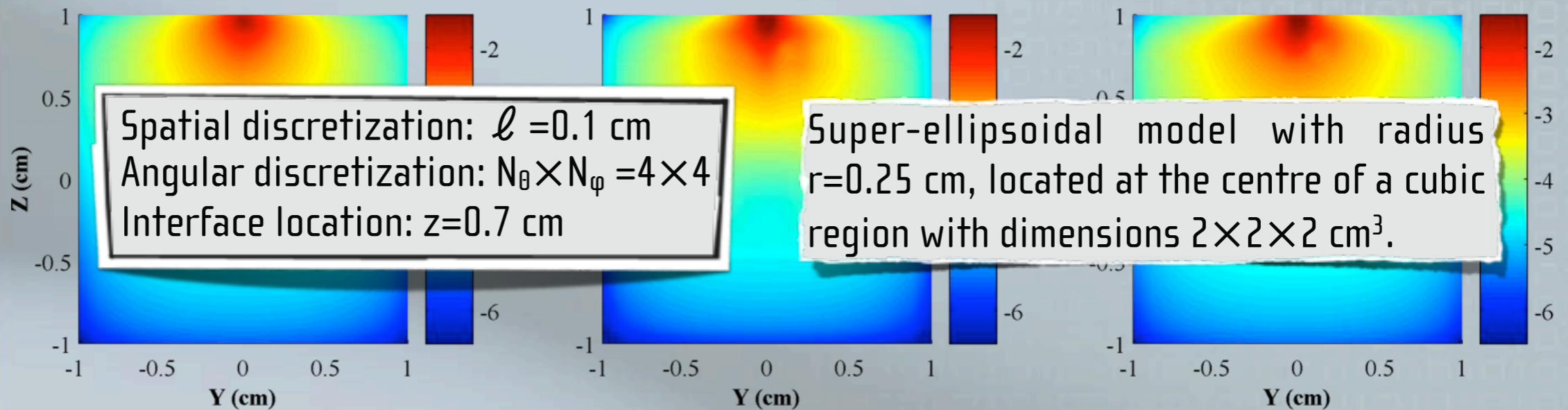
# EVALUATION OF THE STEADY-STATE FORMALISM

## Steady-state results



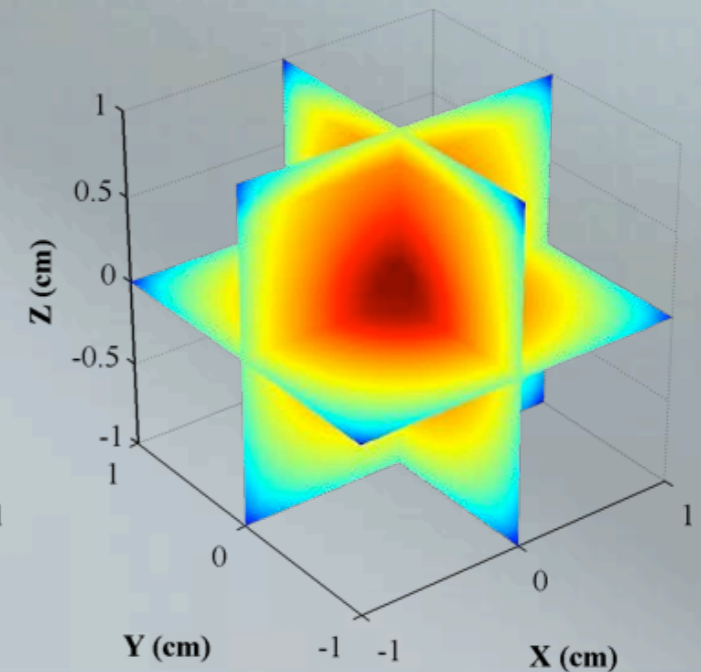
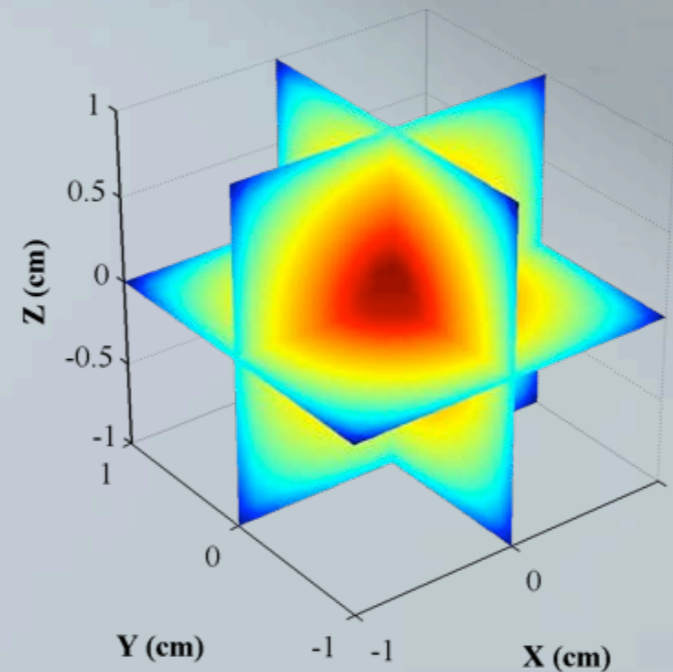
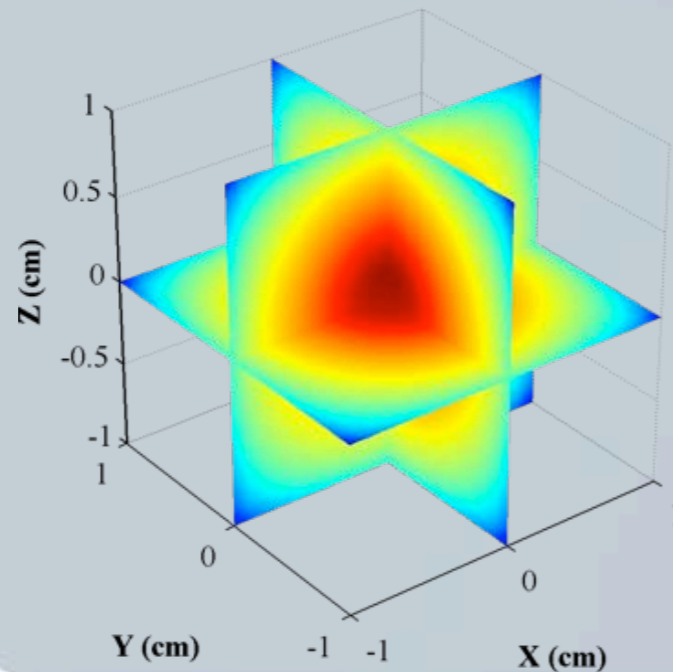
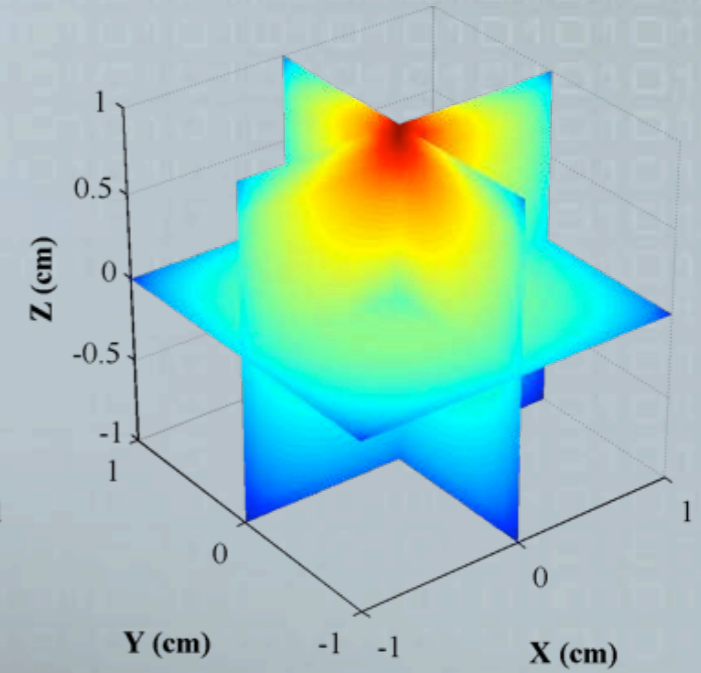
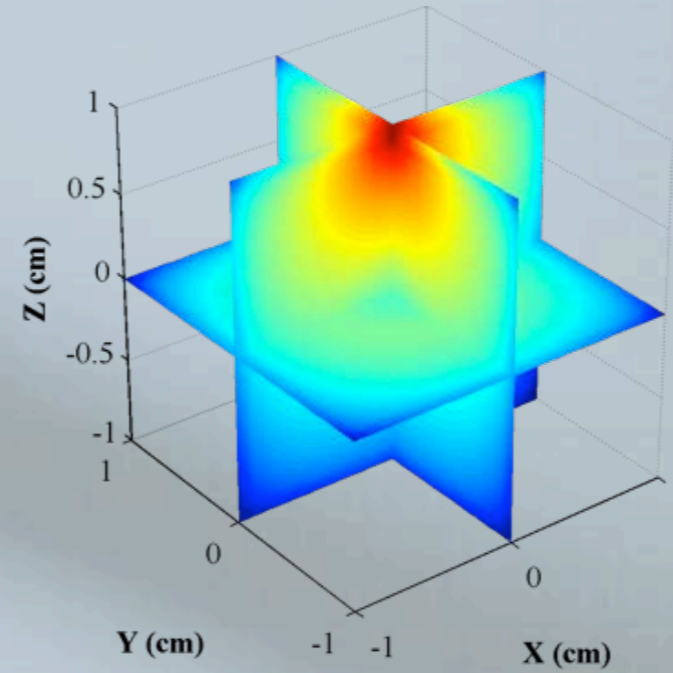
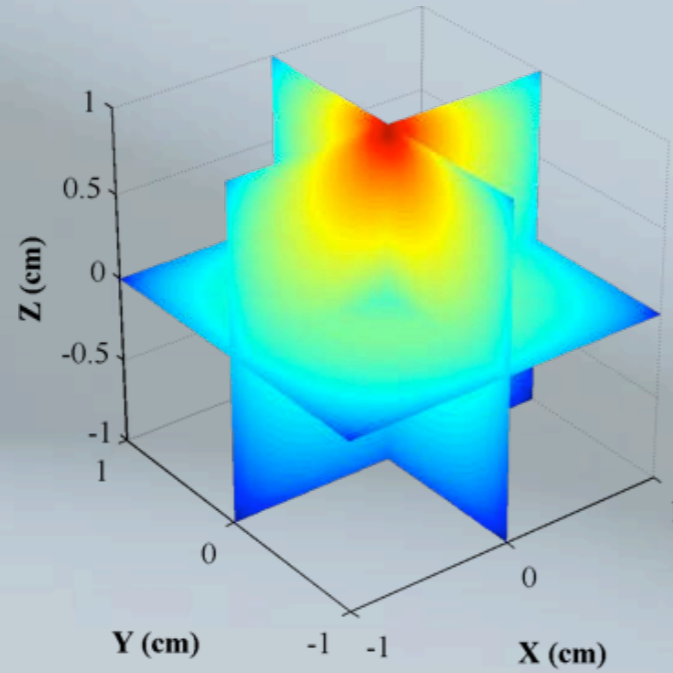
# EVALUATION OF THE STEADY-STATE FORMALISM

## Steady-state results



# EVALUATION OF THE STEADY-STATE FORMALISM

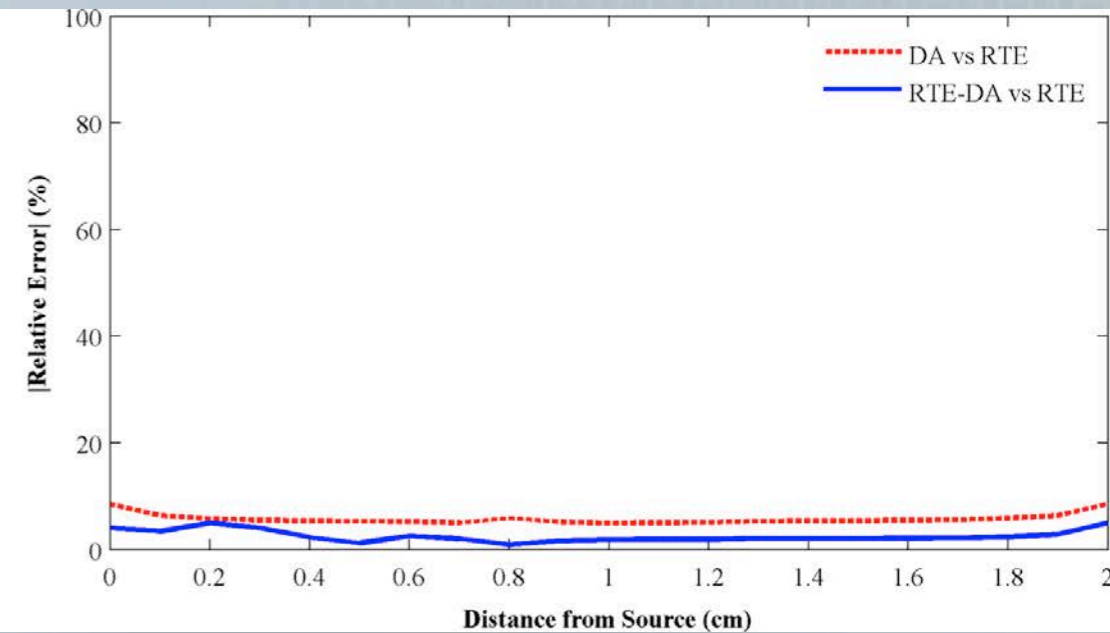
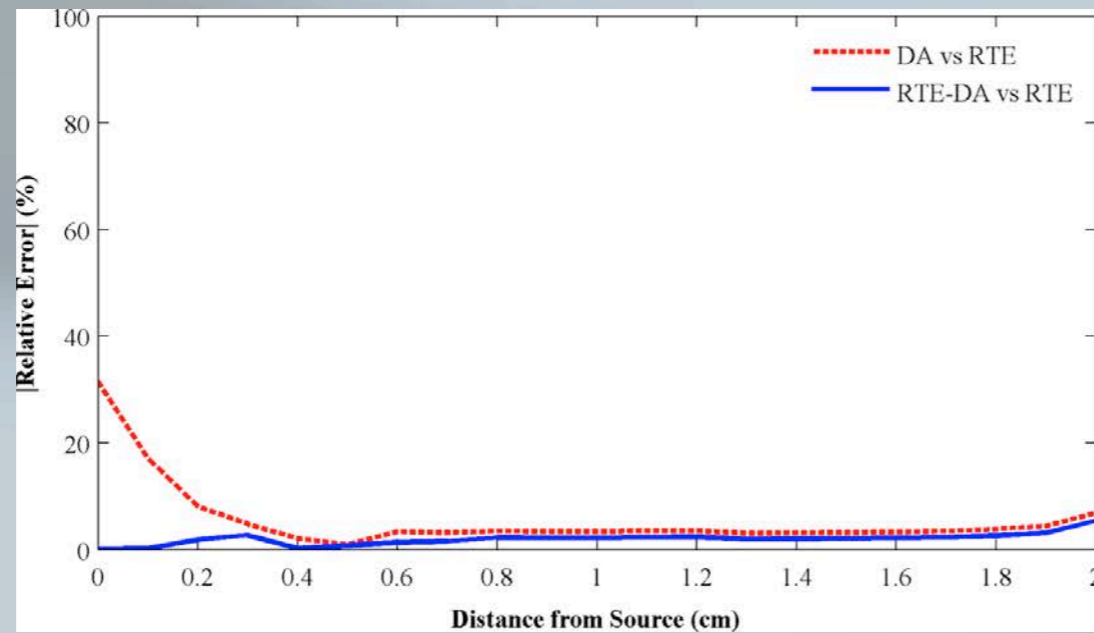
## Steady-state results



# EVALUATION OF THE STEADY-STATE FORMALISM

## Evaluation of the steady-state results

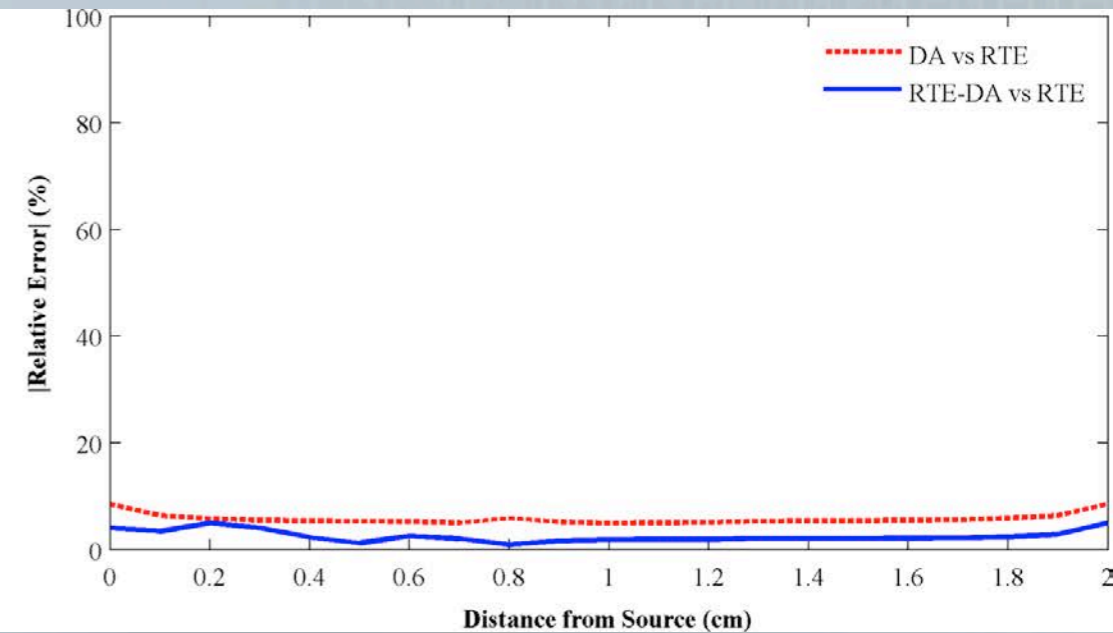
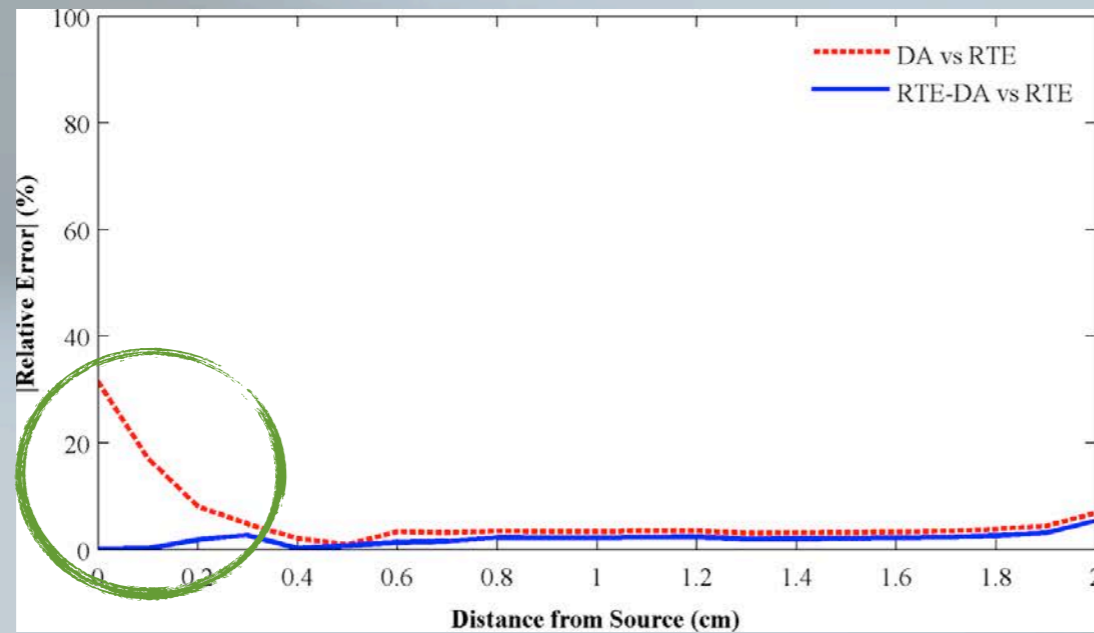
- Accuracy of the method.



# EVALUATION OF THE STEADY-STATE FORMALISM

## Evaluation of the steady-state results

- Accuracy of the method.



	RTE-DA/RTE		DA/RTE	
	Excitation	Emission	Excitation	Emission
Photon Density	~98%	~98%	~94%	~94%

# EVALUATION OF THE STEADY-STATE FORMALISM

## Evaluation of the results on the CW state

- The computational time, the number of iterations and the size of the formulated matrices.

	DA	RTE	RTE-DA
$t_{\text{total}}$ (sec)	29.41	884.58	454.03
$n_x$	81	107	123
$n_m$	76	159	206
$N_{\text{nz}}$	134 864	48 346 456	8 415 584
$N_{\text{total}}$	94 128 804	34 306 448 400	1 867 795 524

# EVALUATION OF THE STEADY-STATE

## FORMALISM

### Evaluation of the results on the CW state

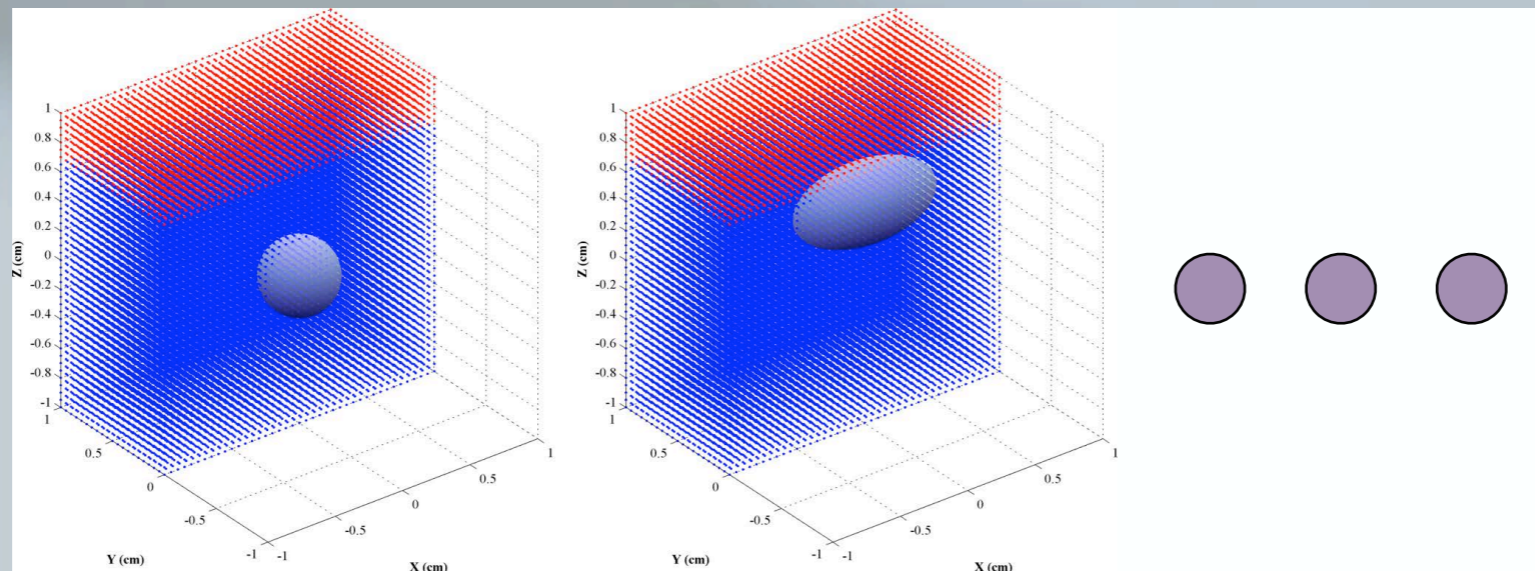
- The computational time, the number of iterations and the size of the formulated matrices.

	DA	RTE	RTE-DA
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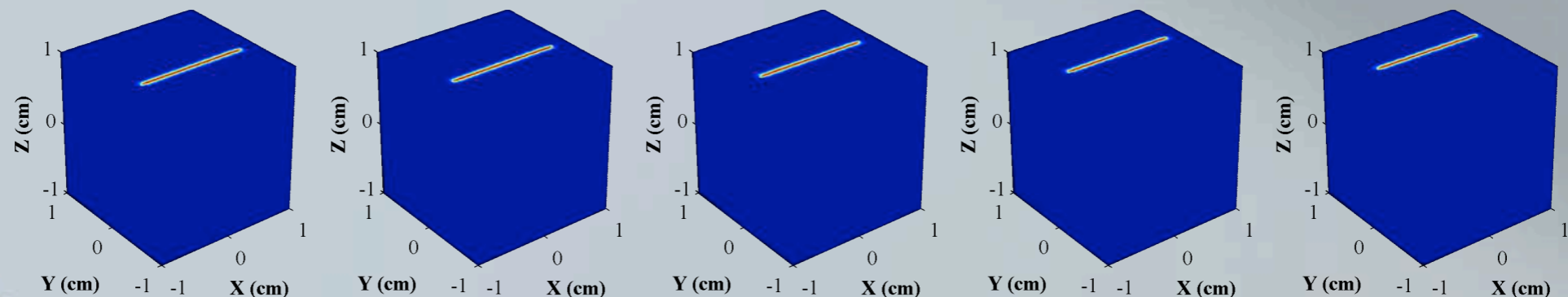
# EVALUATION WITH REAL MEASUREMENTS

## Construction of the database

- > Solution of the forward problem for numerous virtual fluorophores distributions and for every excitation source position.



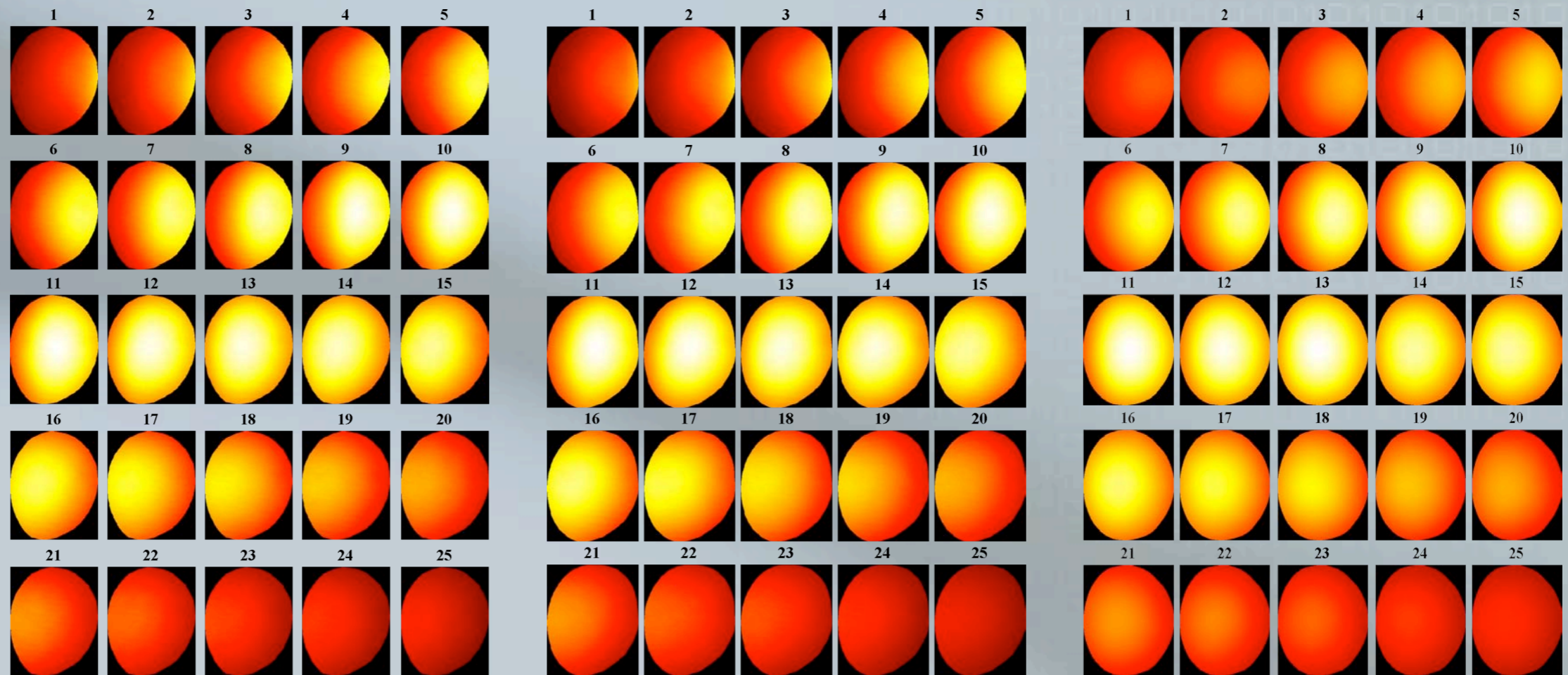
Dual coupled RTE-DA model





# EVALUATION WITH REAL MEASUREMENTS

## An example of measured data



Fluorophore: Alexa Fluor 680.

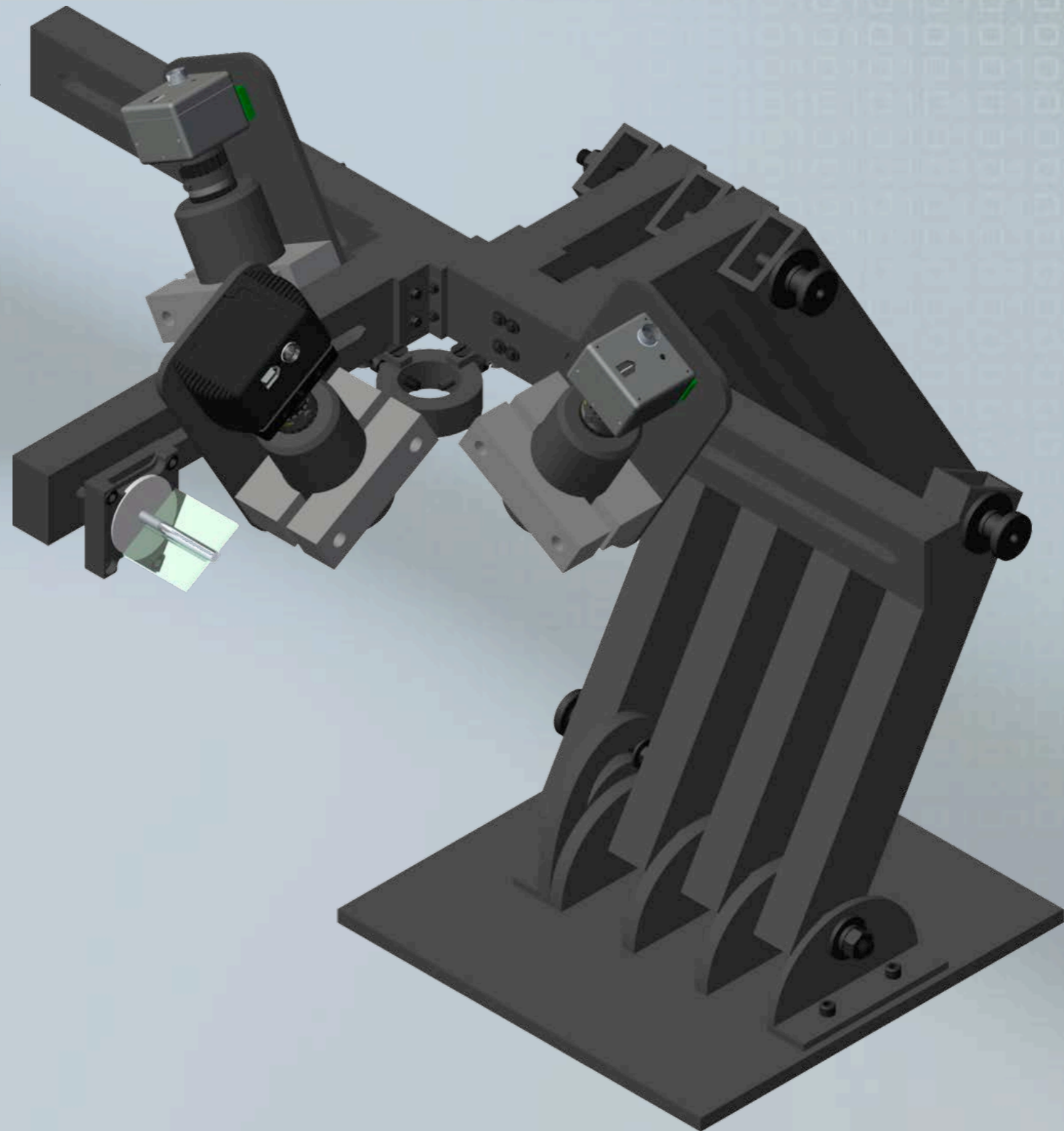
Real geometry: Ellipsoidal with  $d=0.25\text{cm}$  and  $h=0.5\text{cm}$  at  $z=0.6\text{cm}$

Tissue phantom: Suspension of 1% v/v Intralipid and 1% w/v Agarose.

Excitation source: Projected line with scanning step 0.05 cm.

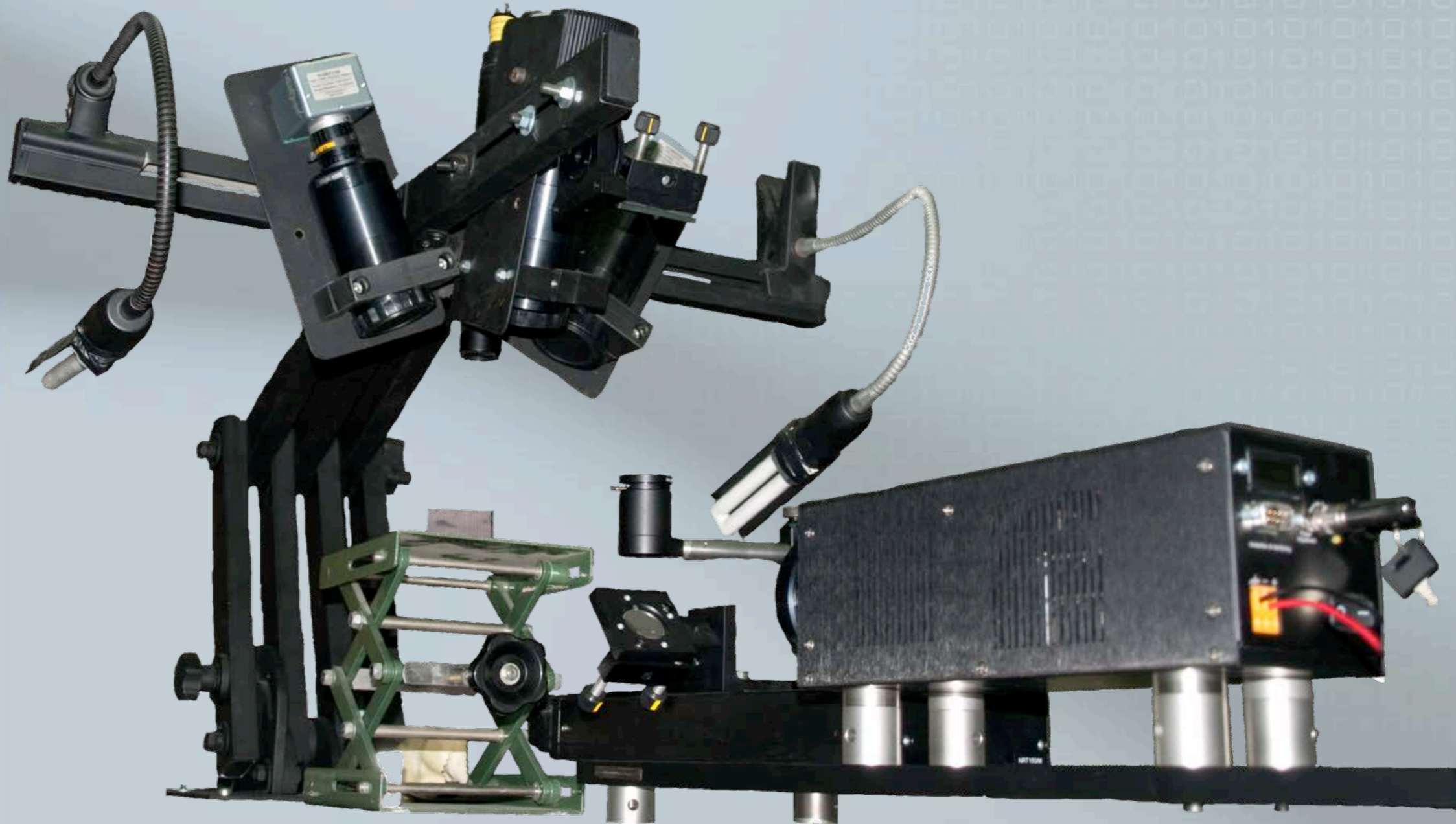
# EVALUATION WITH REAL MEASUREMENTS

The system



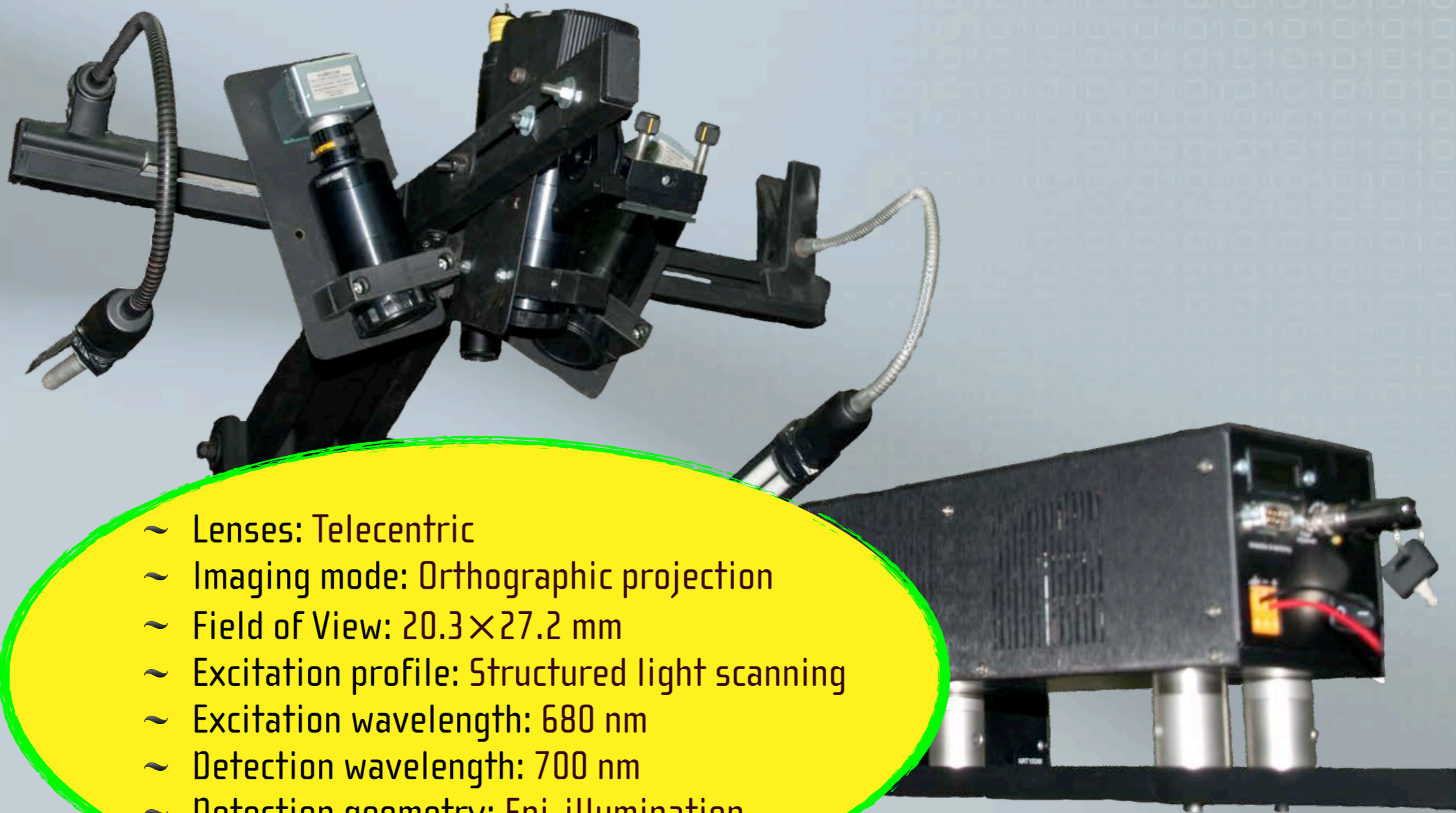
# EVALUATION WITH REAL MEASUREMENTS

The system



# EVALUATION WITH REAL MEASUREMENTS

## The system

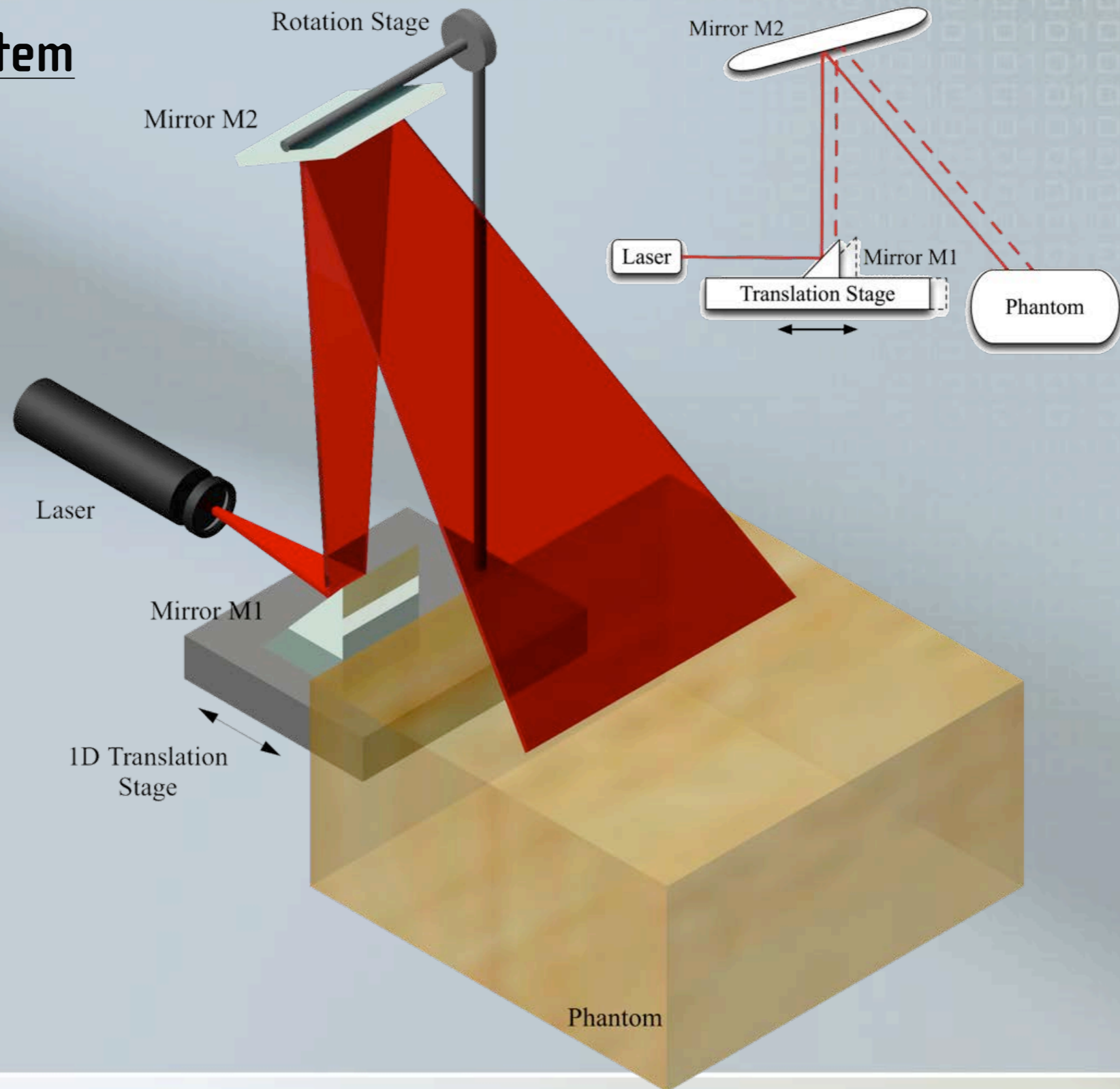


- ~ Lenses: Telecentric
- ~ Imaging mode: Orthographic projection
- ~ Field of View:  $20.3 \times 27.2$  mm
- ~ Excitation profile: Structured light scanning
- ~ Excitation wavelength: 680 nm
- ~ Detection wavelength: 700 nm
- ~ Detection geometry: Epi-illumination



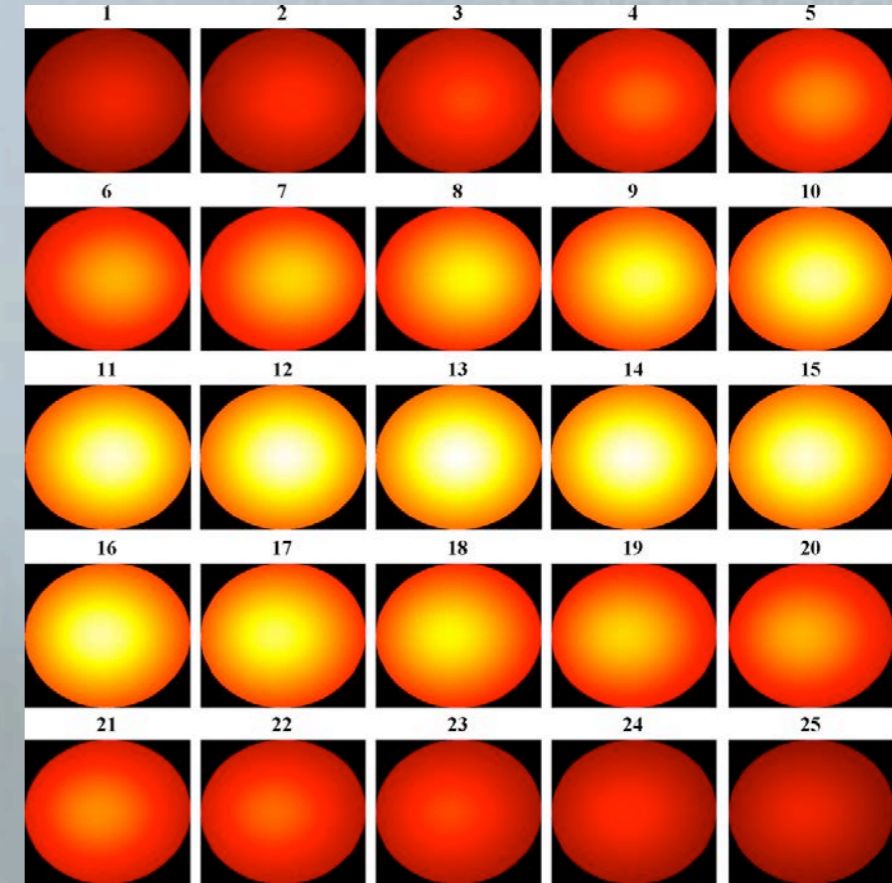
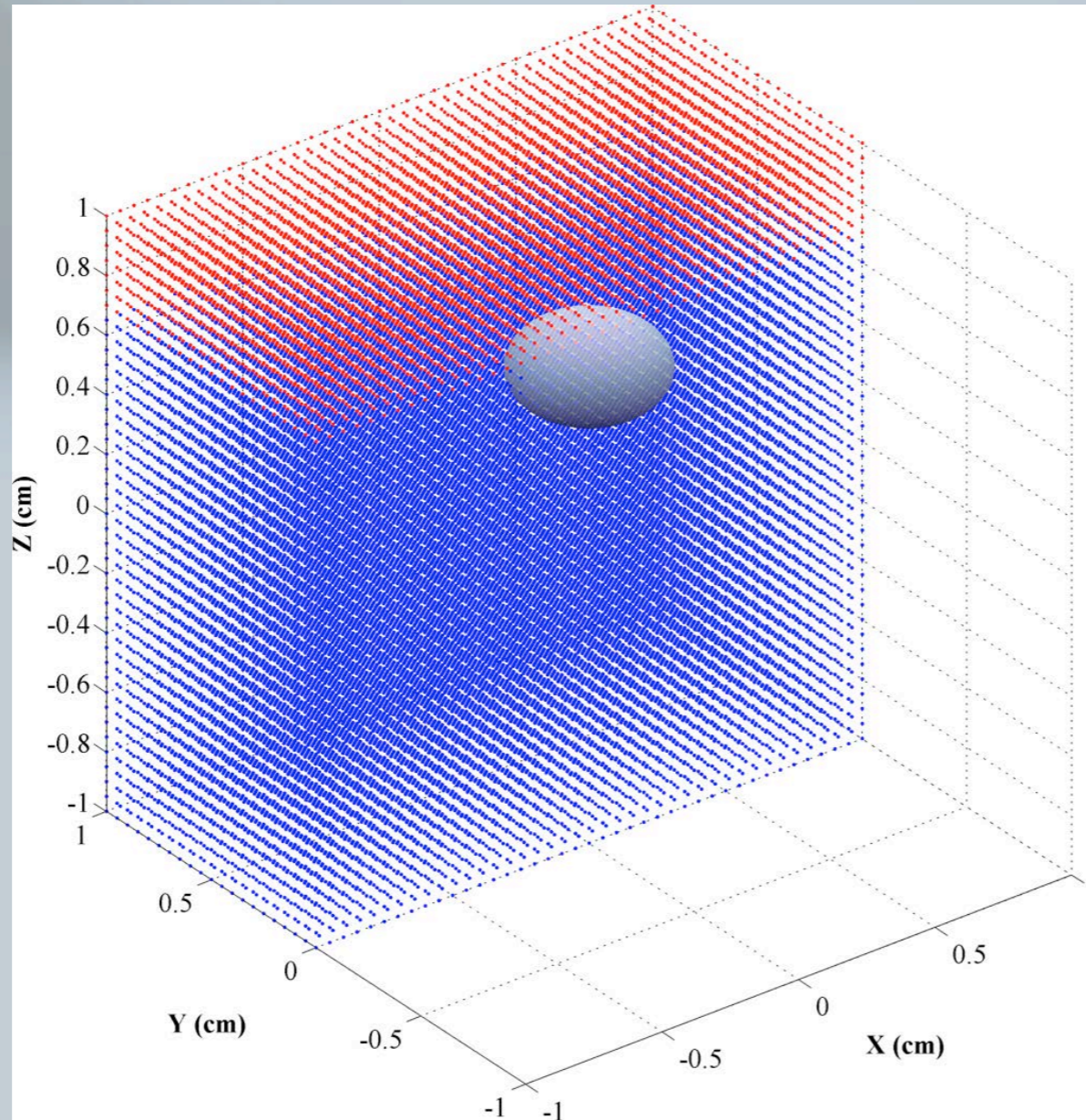
# EVALUATION WITH REAL MEASUREMENTS

## The system



# EVALUATION WITH REAL MEASUREMENTS

The corresponding virtual distribution



✦ The mean accuracy level of the correspondence between real and virtual data exceeds 80%.

# EVALUATION WITH VIRTUAL TISSUE- LIKE MEASUREMENTS

## Evaluation with application on the MOBY phantom

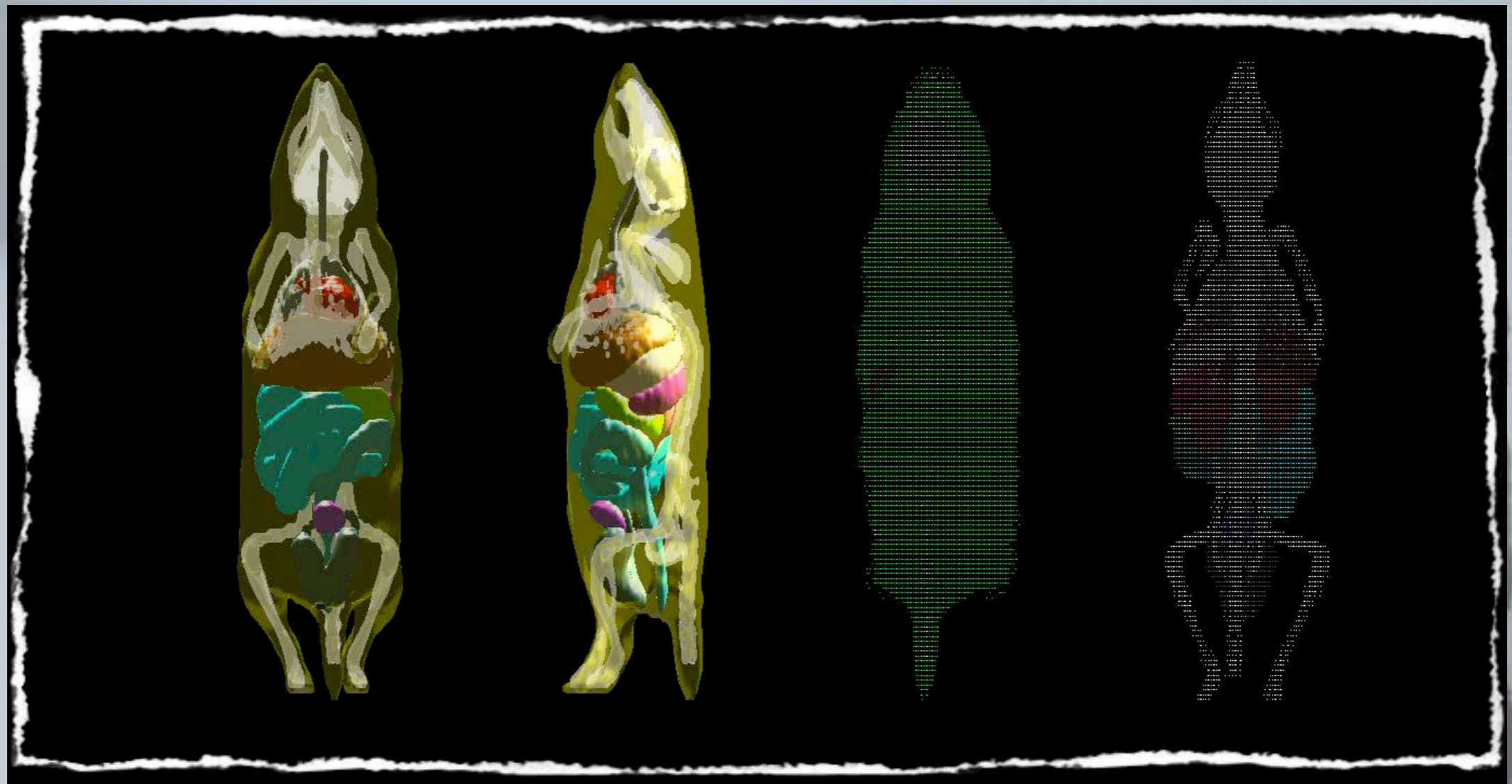


The various organs of this digital mouse phantom have been developed through the utilization of non-uniform rational b-spline (NURBS) surfaces. High-resolution 3D magnetic resonance microscopy (MRM) data, obtained from the Duke Center for In Vivo Microscopy, was used as the basis for the formation of the surfaces. This digital phantom has been utilized for numerous imaging studies, including fluorescence molecular imaging studies.



# EVALUATION WITH VIRTUAL TISSUE- LIKE MEASUREMENTS

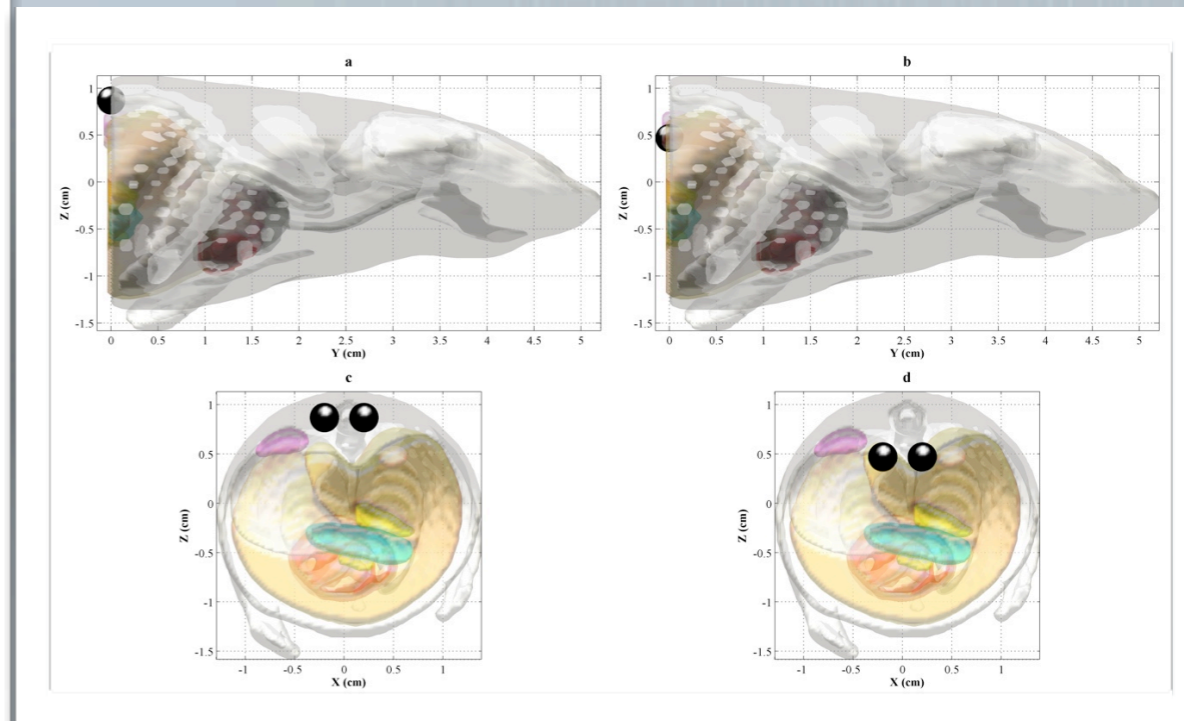
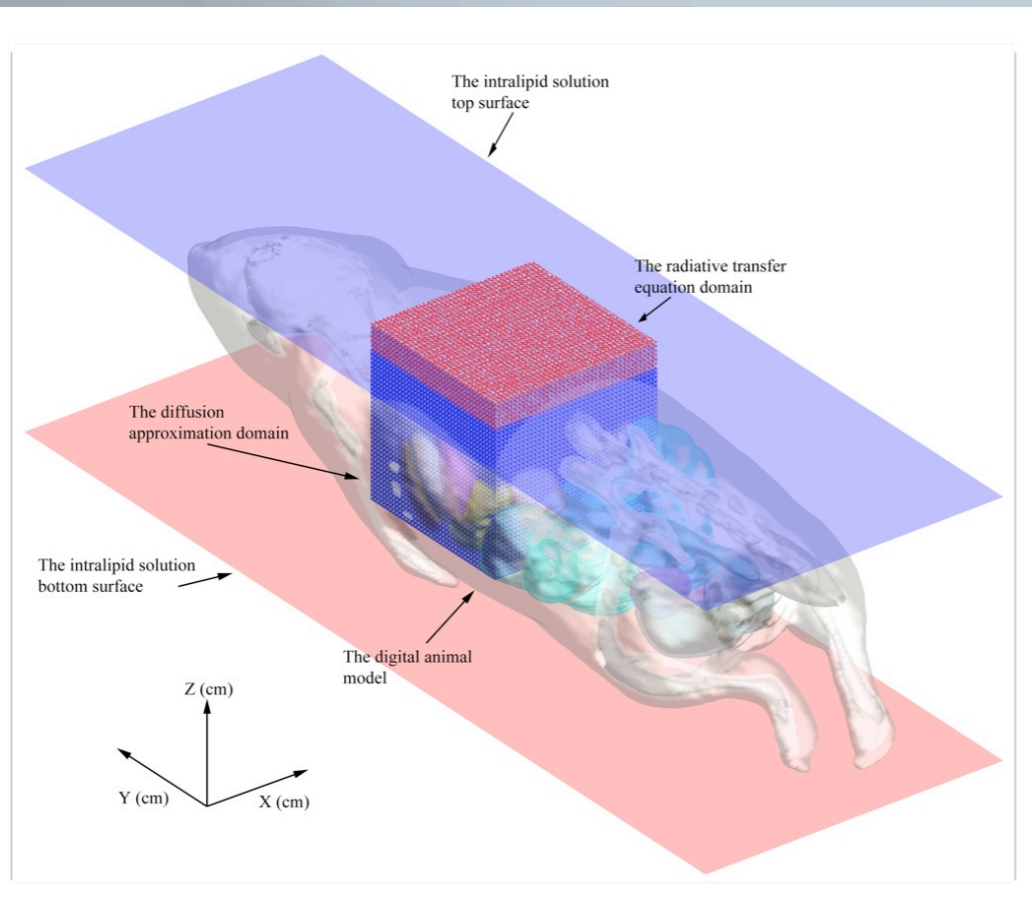
Evaluation with application on the MOBY phantom





# EVALUATION WITH VIRTUAL TISSUE-LIKE MEASUREMENTS

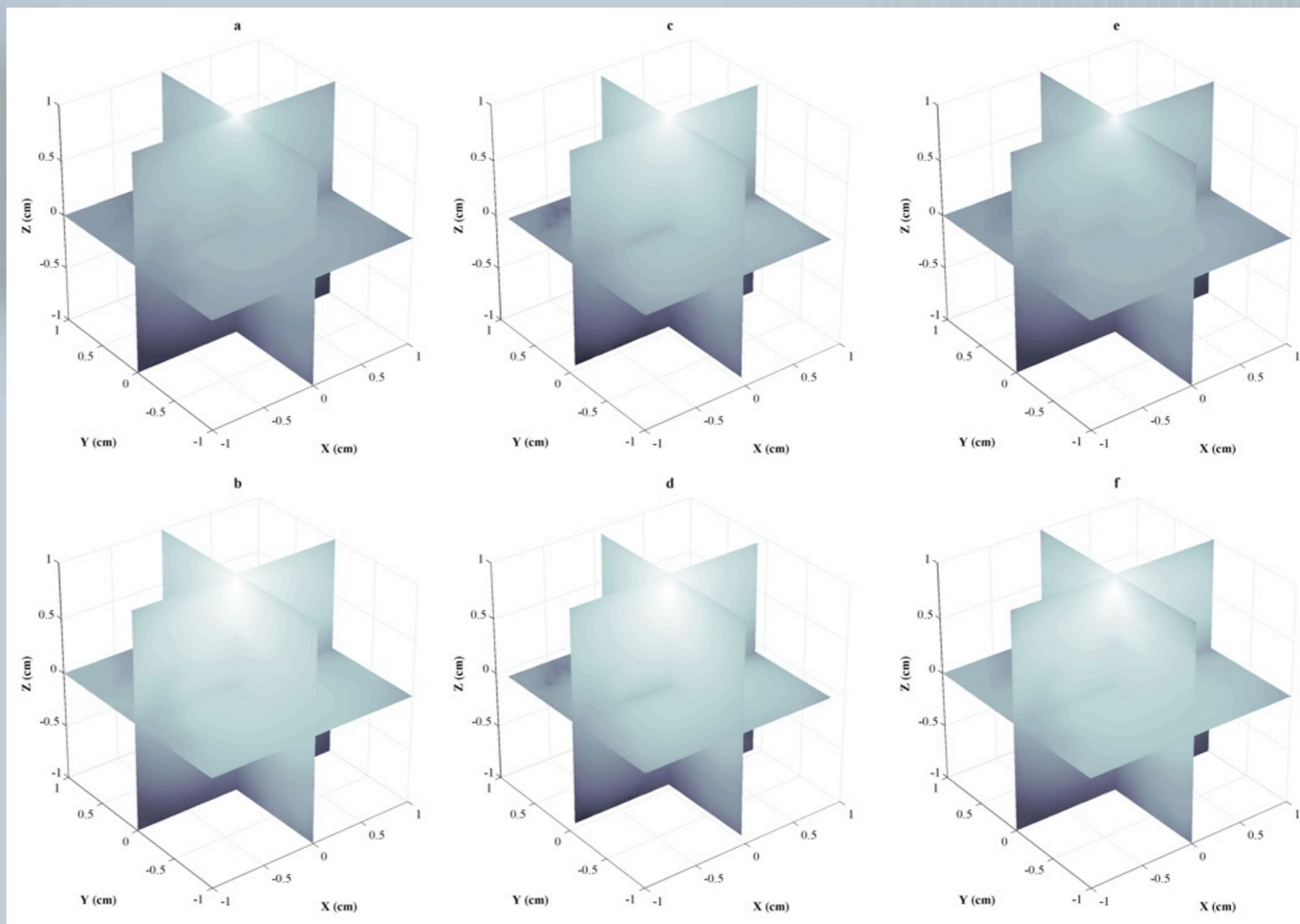
## Evaluation with application on the MOBY phantom



Gorpas D., and Andersson-Engels S., "Dual Coupled Radiative Transfer Equation and Diffusion Approximation for the Solution of the Forward Problem in Fluorescence Molecular Imaging", Imaging, Manipulation, and Analysis of Biomolecules, Cells, and Tissues X, Proc. SPIE, 8225:822522 (2012).

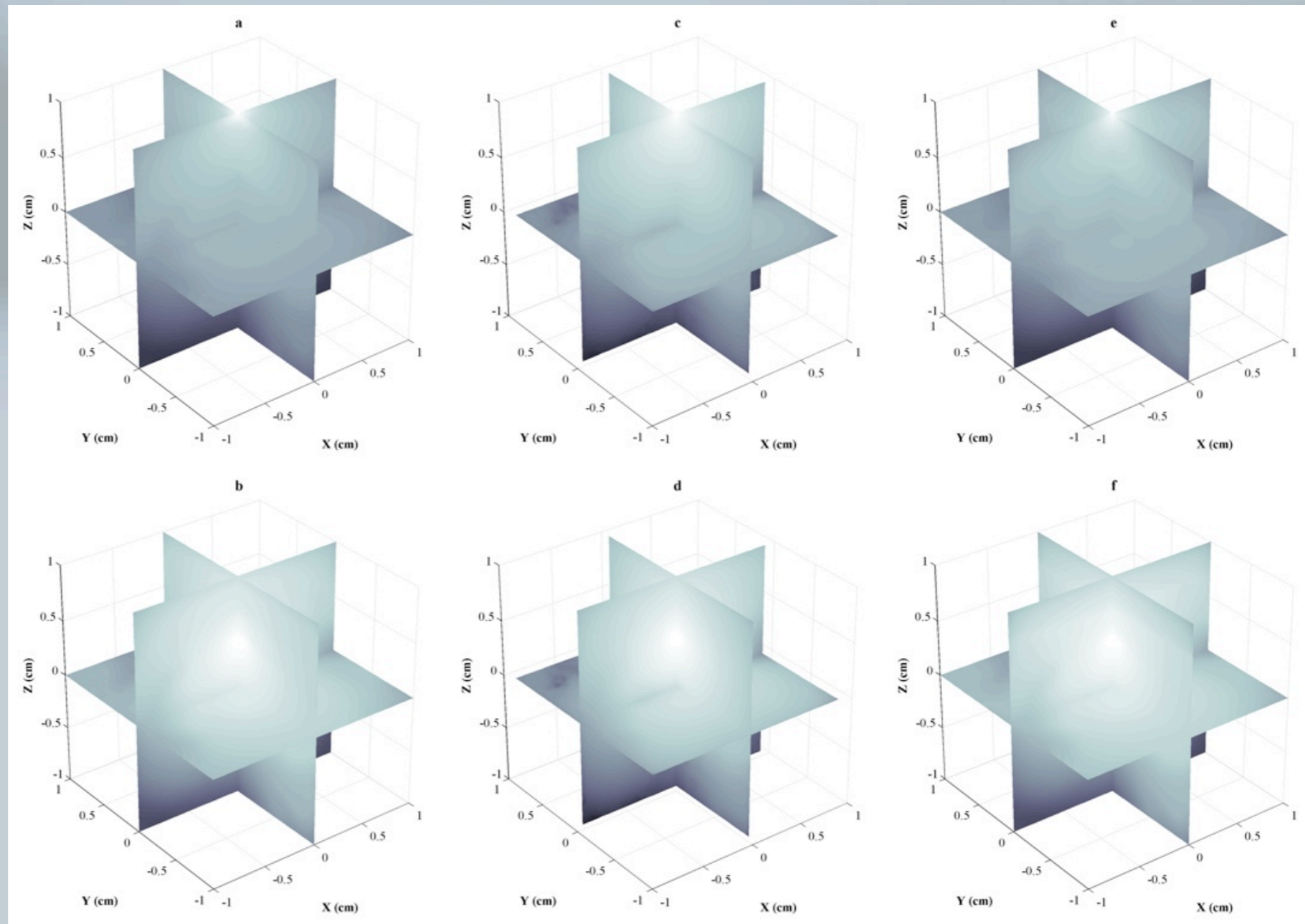
# EVALUATION WITH VIRTUAL TISSUE- LIKE MEASUREMENTS

Evaluation with application on the MOBY phantom



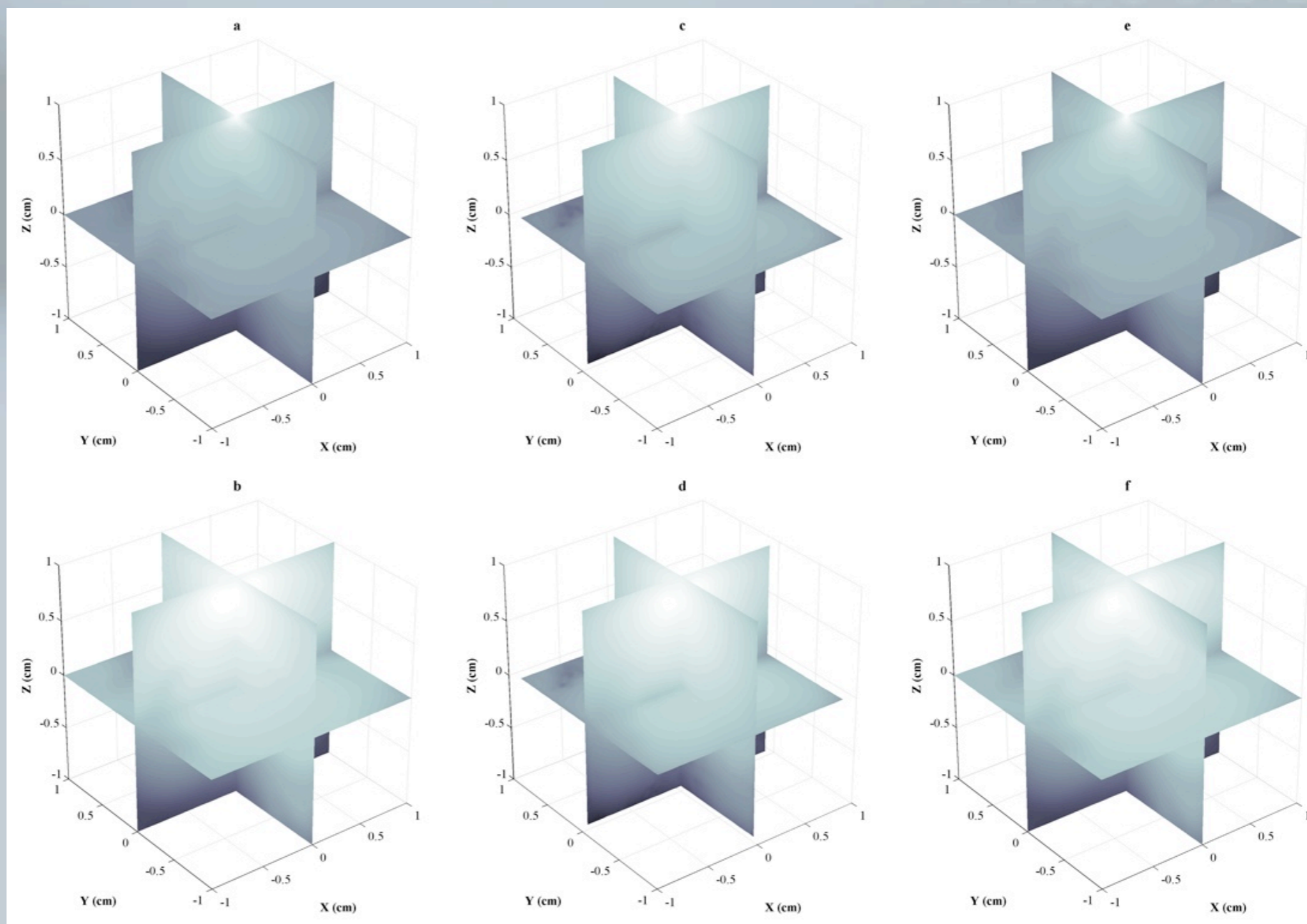
# EVALUATION WITH VIRTUAL TISSUE-LIKE MEASUREMENTS

Evaluation with application on the MOBY phantom



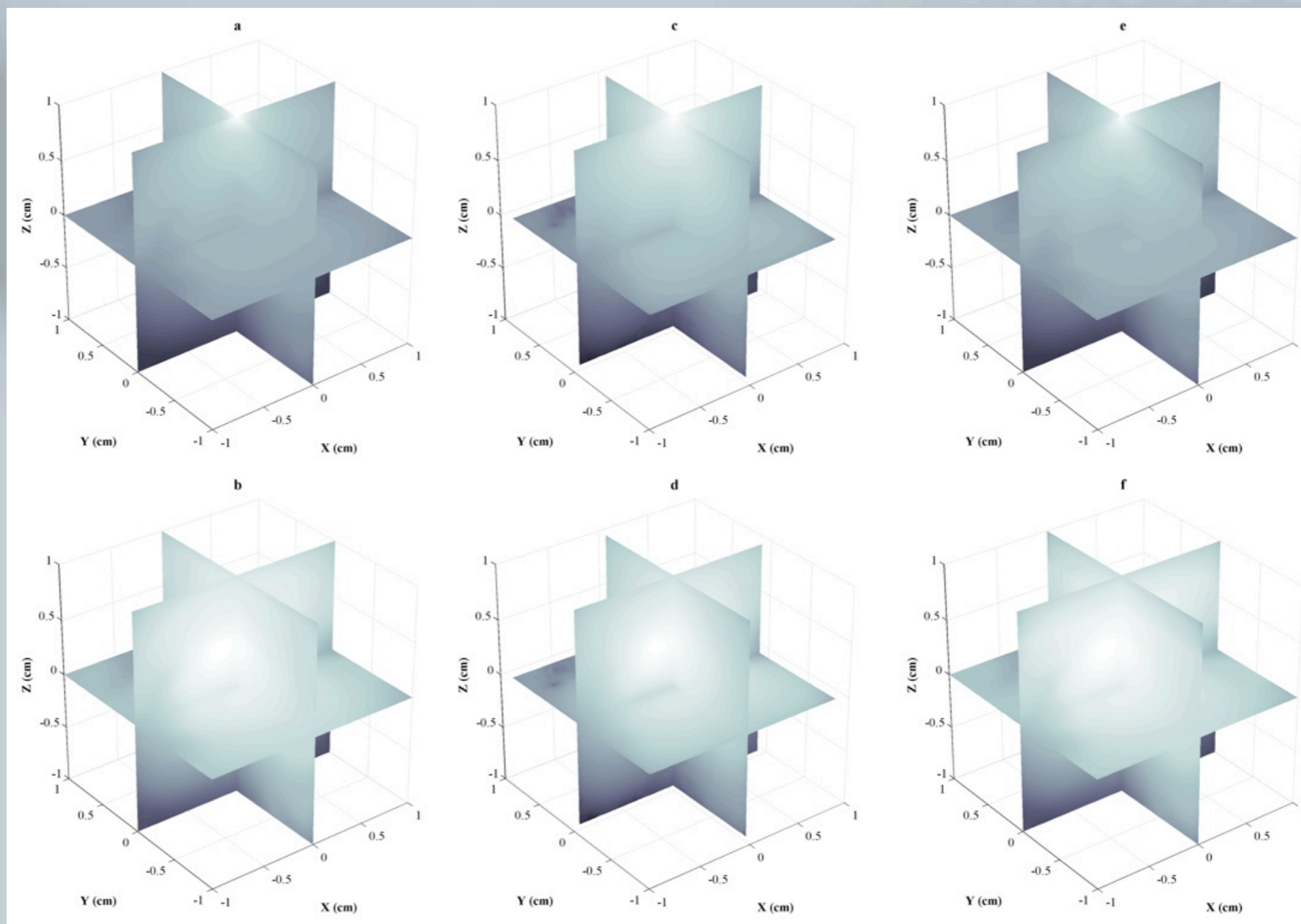
# EVALUATION WITH VIRTUAL TISSUE-LIKE MEASUREMENTS

Evaluation with application on the MOBY phantom



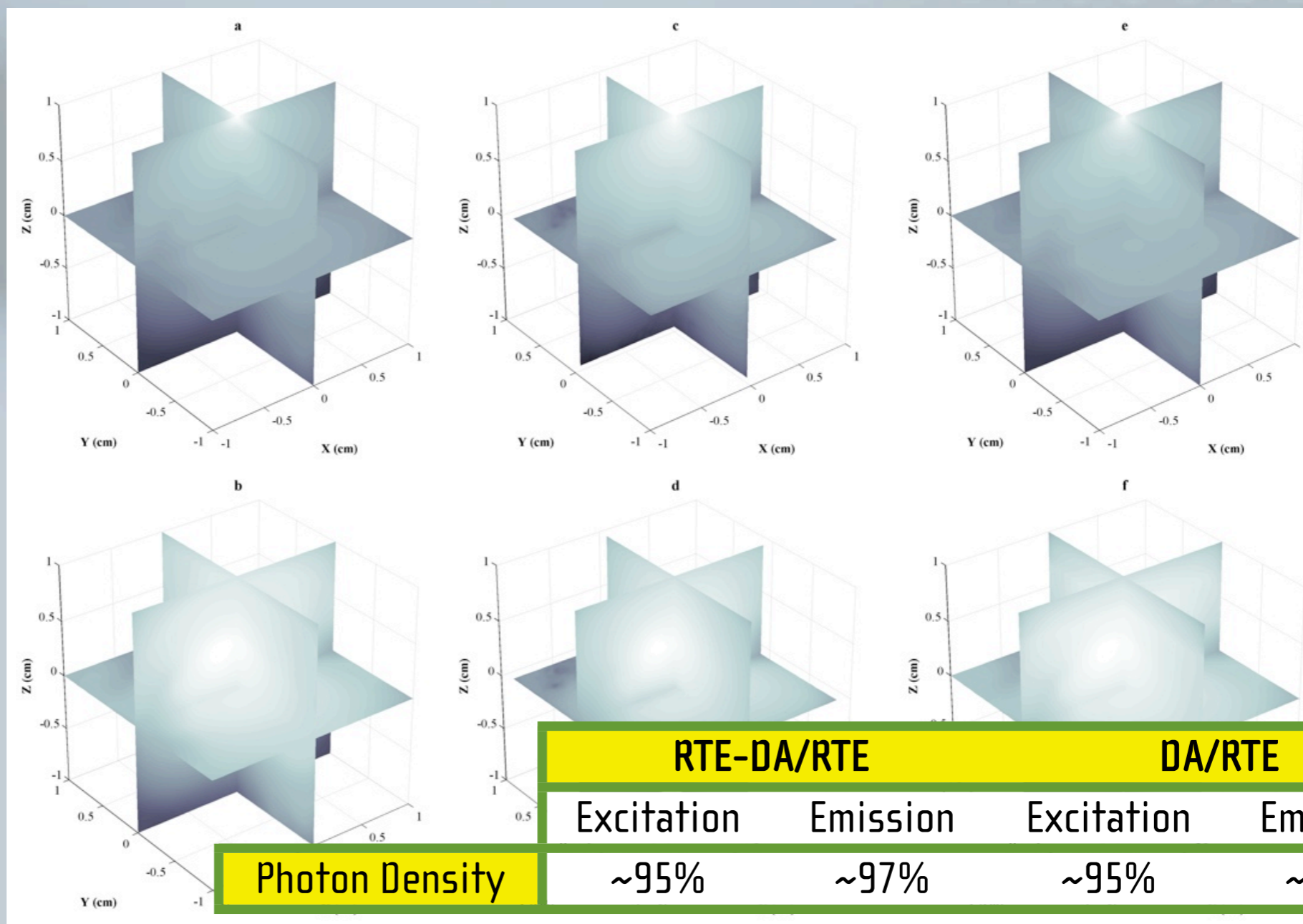
# EVALUATION WITH VIRTUAL TISSUE-LIKE MEASUREMENTS

Evaluation with application on the MOBY phantom



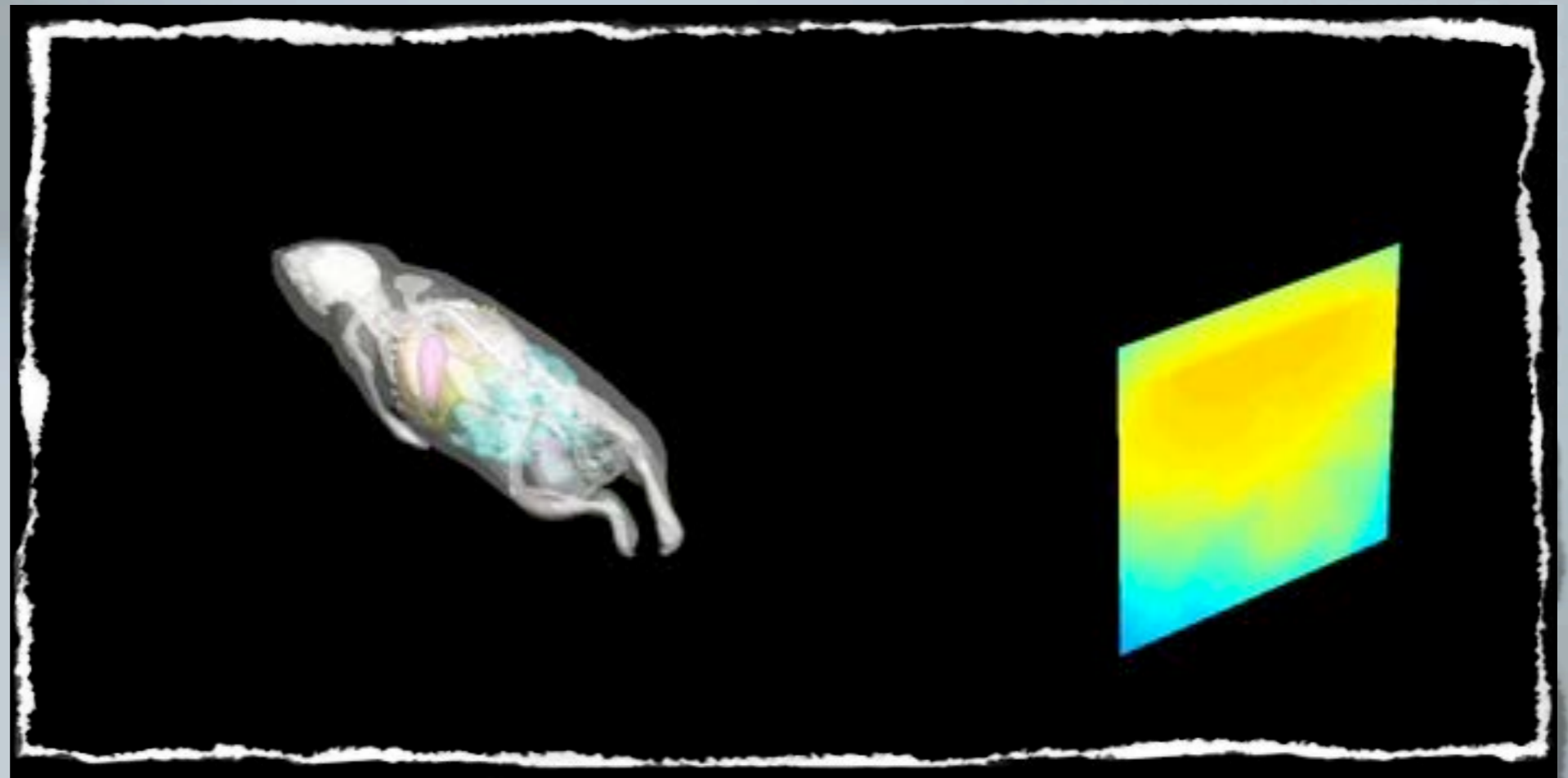
# EVALUATION WITH VIRTUAL TISSUE-LIKE MEASUREMENTS

Evaluation with application on the MOBY phantom



# EVALUATION WITH VIRTUAL TISSUE- LIKE MEASUREMENTS

Evaluation with application on the MOBY phantom



# OUTLINE

The fluorescence phenomenon.

Fluorescence molecular imaging - the problems.

Image acquisition systems.

Light propagation models.

Conclusion.





# CONCLUSION

## Contribution of the research

- Solution of the forward problem in fluorescence molecular imaging with utilization of the RTE model.
- Solution of the forward problem in fluorescence molecular imaging with the dual coupled RTE-DA model.
- Development of an epi-illumination fluorescence molecular imaging with scanning structured excitation source.
- Fluorescence acquisition with angular information.



# CONCLUSION

## Ongoing research

- Development of an inverse problem solution, based on super-ellipsoidal models and the Levenberg-Marquardt technique.
- Further evaluation of the dual coupled RTE-DA model on non-homogeneous synthetic phantoms.
- Optimization of the algorithms for time efficacy increase.



# CONCLUSION

## Possible prospects

- Optimization of the spatial and angular discretization schemes.
- Investigation of the possibility to apply the RTE based forward solver.
- Study of the molecular information of the biomarkers.
- Development of small animal tomographic applications.
- Adaptation of the methodology for breast cancer detection.
- Development of a compact and portable system!



# CONCLUSION

Is the solution of the reconstruction problem the border of this field?



Thank you for  
your attention!!!

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