Online task distribution simulation in smart factories

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***Index Terms*—Decision Support System; work schedule re-adaptation; Industry 4.0; task scheduling; resource allocation; optimization; simulation.**

I. INTRODUCTION

Resource planning and management is a critical issue faced by modern factories and enterprises. The process of assigning tasks to the appropriate machinery and human resources in order to produce an efficient work schedule can be time-consuming [1]. The challenge in resource management lies also in the discovery of an optimal solution that would minimize costs, meet employees preferences, distribute shifts fairly among employees, and satisfy various other constraints. Furthermore, there are cases when the work schedule has to be re-adapted due to events that may occur on the shop floor, such as the arrival of a new task or a malfunction in equipment being operated. To this end, Decision Support Systems (DSS) [2] can contribute to the creation and automated update of a work schedule in addition to the discovery of efficient assignment of the proper resources, in order to reduce costs, meet the production goals and achieve high levels of employees satisfaction.

For solving the resource management and allocation problem, many assignment and decision-making approaches using different criteria have been proposed in the literature. The authors in [3] present a human resource assignment tool for human to tasks assignment, which includes two possible methods, the competency-based, and the preference-based method, using the competence levels and the preferences of each employee, respectively for the task allocation. Another study [4] proposes a multicriteria decision-making methodology for the human resource assignment problem. The method proposed in that work takes into account the workers sensitivity to risk, in order to maximize the global carefulness and minimize the global cost and dislike. In [5], the authors propose a mixed integer programming model which aims to find optimal job rotation schedules in work environments characterized by manual tasks. However, in that work dynamic parameters of worker condition were not included.

In study [6], a human resource assignment method is presented, which takes into account both static and dynamic characteristics of employees, and calculates the assignment cost for each employee in terms of capability, workload balancing and impact on the shop floors work schedule, in order to select the solution of the minimum cost. In another relevant recent work [7], a Human Resources (HR) optimization engine that is able to schedule maintenance tasks in real-time and assign them to workers, is described. The locations of workers and the locations of tasks are considered in that work, in order to take distance into account.

Proper configuration of a DSS is a complex task, as the companys policies have to be included to the system. Apart from the methodologies and real-time HR optimization systems, simulation tools are also incorporated into DSS. Study [8] presents a simulation-based DSS for task scheduling and production line planning. The system proposes an automated simulation model for the creation of the initial work schedule and its re-adaptation according to the events occurred. In study [9], the authors describe a simulation-based method for a DSS aiming at the evaluation of the operational procedures taking place on the shop floor. In addition to simulating and developing a work plan, some systems include methods that can handle new events coming on the shop floor. The authors of [10] propose a hybrid decision-making system for resource allocation and re-adaptation in software project management, using dynamic characteristics of workers and tasks. In case of any change on one of these characteristics, the system is able to evaluate the current status and make the proper adaptations in order to produce an optimal schedule.

In literature some of the DSS include also web interfaces in order to visualize all the needed information. In [11], the authors provide a review of web-based simulation tools, and highlight the advantages and disadvantages of each system. The authors of [12] have implemented a web-based simulation tool aiming at the creation of job rotation schedules for human-based assembly systems. The proposed tool is responsible for the visualization of the simulations output and the presentation of tasks executed in the production line.

This paper presents a feature-rich Task Distribution Engine (TDE), which is an online/offline, real-time decision-making component that aims to optimize workers impact on daily tasks in the workplace, as well as to distribute tasks efficiently. Its main objectives are the assignment of tasks to resources, such as human resources and machines, and the adaptation of the factorys procedures and work schedule by monitoring the status of tasks and resources, in order to optimize productivity, decrease response times and deal with unexpected events. The main functionalities of the TDE, among others, include the assignment of tasks to resources according to multiple criteria, such as skills, capabilities, and preferences. In addition, TDE supports workload balance monitoring of employees of the same team in order to perform task reassignments for improving workload balancing. The simulation mode gives managers the ability to perform multiple simulation runs with different configuration each time, in order to discover the best-suited parameters to their preferences and needs. Lastly, the visualization interface of the TDE displays information regarding the tasks execution and the work schedule of each shift via an intuitive interface.

The main contribution of the paper concerns the development of a task scheduling and assignment system with the following characteristics:

- Generic model applicability (i.e. humans, factorys machines, components, and other resources, which are comprised of heterogeneous variables).
- Adaptivity in break outs and resilience.
- Multi objective, multi constraints optimization.
- Real-time and simulation modes, allowing both the resource planning and evaluation of different scenarios that can occur on the shop floor, respectively.
- Comprehensive and interactive visualization interface for decision support.

To the best of our knowledge, the support of both real-time and simulation modes, alongside with the implementation of a web-based visualization interface for user interaction and the high configuration capability, make the Task Distribution Engine (TDE) a state-of-the-art task assignment and scheduling system.

The rest of the paper is organized as follows: Section 2 presents the definition of the data models used by the TDE. In section 3, the systems architecture and functionalities are presented. The simulation results derived from the case study are demonstrated in section 4. Lastly, in section 5, the main conclusions are summarized.

II. Data Models

The two types of entities that are handled by the TDE are the tasks (T) and the resources (R). Each entity type has its own data model (M), which is a set of pre-defined variables (V). Each resource type is defined by a specific data model based on its type. A variable included in each model can belong to one of the following three sets: V_s (static variables), V_d (dynamic variables), and V_{comp} (composite variables). A static variable does not depend on time, while a dynamic variable can change over time. A composite variable is a special type of dynamic variable, which is computed on the fly by taking into account the values of two or more other variables of the model, which can be static or dynamic.

TDE utilizes data models, such as task model M_T, worker model $M_{R\ (WORKER)}$ and machine model $M_{R\ (MACHINE)}$ in order to provide a resource management solution. Those models are used as input for the evaluation of possible different assignments in order to discover the most effective one, according to the needs and requirements provided.

III. System Description

The pyramid model presented in Fig. 1 displays the Task Distribution Engines components. At the bottom of the pyramid, resides the the Decision Support module, which is responsible for the implementation of the decision-making algorithm and export of the output. At the top of the pyramid resides the web-based Visualization interface, which is the user-interface and is responsible for the visualization of the Decision Support modules output and the interaction with the end user, who usually is a manager. The Decision Support module provides a REST web services interface [13], presented in the middle of the pyramid, in order to exchange data with other components in JSON format. The Web-based Visualization interface utilizes the web services in order to communicate with the Decision Support module. The following sections describe elaborately the the functionalities of the TDE components.

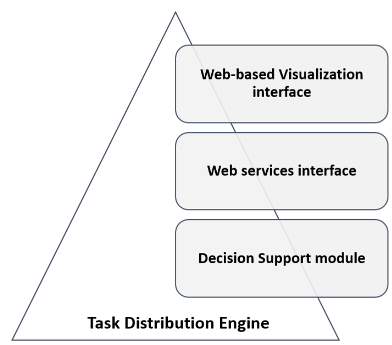

Fig. 1. Task Distribution Engines components.

A. Decision Support System module

The Decision Support module supports two different operation modes, the real-time and the simulation mode. In real-time mode, the engine acquires information from a database by making appropriate requests periodically or when required, via a background process. In simulation mode, in which this paper focuses on, the user can insert the desired input data and settings, and review the changes made in the work schedule during the simulation run. The main objective in this case is to allow the user to run different scenarios using different settings. In particular, in order to perform a simulation run, the initial state, which includes the scheduled tasks assigned to workers and machines, has to be defined and loaded. The user then defines the events that will be created during the simulation run. Events are any changes occurring on the shop floor, such as the arrival of a new task or a change regarding the availability of a resource (worker/machine). For each event, the type, the content and the arrival date-time have to be set. The engines output includes the applied re-adaptations in the work schedule, in order to be reviewed by the manager. The main functionalities of the Decision Support module are presented next.

a) Task prioritization: In general, task prioritization is a complex job for a manager due to the considerable number of different tasks with diverse requirements that have to be performed. The TDE provides automated task prioritization by computing a priority score for each task, the Task Priority Weight (TPW), which is a value in the range of (0, 1] and it is used as a parameter in order to schedule tasks. It is worthwhile to mention that priority weights can also be translated into distinct priority levels, such as low, medium, high, etc. depending on the organizations needs. Priority level, complexity score, and interruption attributes are the task characteristics taken into account when computing the TPW. Equal weights are assigned to the first two parameters, which are first normalized in [0, 1], in order to compute the TPW. The TPW is incremented by 10% in case the task is not interruptible.

b) Scheduling: According to TDE operation, scheduling refers to both the creation of the initial work schedule and the realization of any re-adaptation of it throughout the shift, based on events occurring on the shop floor. TDE is responsible for the analysis of the requirements of a new event, in order to re-adapt the current work schedule if required. In case of a new task entered in the work schedule, TDE receives the needed information from the task description, which contains information such as the task's type, the estimated duration, the type of machine required (if any), the trade of the employee required and other variables. TDE proposes the best-found solution by evaluating different possible assignments. A candidate solution includes the starting time of the new task, the selected worker(s) and machine(s) for the task. Additionally, it includes other modifications in the work schedule that are necessary to be applied due to the insertion of the new task. Alongside, a score is being calculated for each solution, which is used to evaluate its suitability.

In order to evaluate and rank the different possible solutions, a constrained optimization [14] method is applied to solve this multi-objective decision-making problem. Constrained optimization is the process of optimizing an objective function with respect to various variables where constraints are applied. The objective function is either a cost function, which must be minimized, or a utility function, which has to be maximized. Constraints can be either hard constraints, which are required to be satisfied or soft constraints, which are preferred but not required to be satisfied.

The problem in this study is formulated as follows. Let C_i^H. and C_j^S denote a hard and soft constraint i and j, respectively. Hard and soft constraints are functions which can have the following values: $C_i^H \in \{0,1\}$, $C_i^S \in [0...1]$. Therefore, in this work, it is assumed that a soft constraint can be violated, be fully or partially satisfied. The utility function to be maximized is the following,

$$U(x) = \begin{cases} 0, if \sum_{i=0}^{h} C_i^H < h \\ \sum_{i=0}^{h} C_i^H + \sum_{j=0}^{s} C_j^S \end{cases} \tag{1}$$

where h is the number of hard constraints and s is the number of soft constraints considered. From the definition above it can be deduced that the utility function U(x) is actually the sum of functions where each one refers to the conformance to a soft constraint, in the case when all hard constraints are satisfied.

Simply summing the number of the fully and partially satisfied constraints in order to compute the utility function, and by extension, the solution score, may not be an optimal approach, as some constraints may be more important than others. Therefore, a Priority Level (PL), which denotes the importance of a particular soft constraint, is defined for each soft constraint, and it can be one of the following values: 1, 2, or 3. As a result, the final form of (1) is the following (2),

$$U(x) = \begin{cases} 0, if \sum_{i=0}^{h} C_i^H < h \\ \sum_{i=0}^{h} C_i^H + \sum_{j=0}^{s} \left(PL_j \times C_j^S\right) \end{cases} \tag{2}$$

where m is the number of different candidate solutions s that are evaluated.

The selected solution S_{sel} is:

$$S_{sel} = max\left(U_s\left(x\right)\right) \;\; for \;\; s = 1, \ldots, m \tag{3}$$

After the evaluation of different possible solutions, the solution with the highest score is being selected and the work schedule is being updated. The Decision Support module is responsible also to make any adaptations in cases of tasks times overlap. If the starting time of a new task overlaps another tasks starting time, the pre-scheduled starting time of an already scheduled task changes. Fig. 2 depicts the steps followed by the TDE, when a new task is inserted into the work schedule. Task prioritization is utilized in order to set the scheduled starting time of a task. The earliest starting time possible for a new task is determined based on the TPW computed per each task. Then the hard constraints are checked in order to eliminate resources based on their capabilities. At the next step, the score of each remaining solution is calculated via the evaluation of the soft constraints considered.

B. Web-based Visualization interface

Task Distribution Engine is accompanied with a web-based Visualization interface, connected through services to the Decision Support module, as already mentioned in this section. This interface aims at visualizing the initial work schedule, the

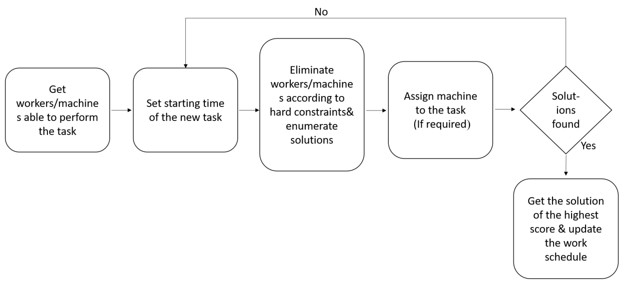

Fig. 2. Steps of the real-time task scheduling approach.

tasks to workers and machines assignment, and any change or re-adaptation occurred in the work schedule. The Web-based Visualization interface provides a user-friendly interface presenting all the necessary information to the user, allowing a more straightforward interaction with the system without the need for any expert knowledge.

The information visualized by undirected network graphs, implemented with VIS library [15], concern the resources allocation (Fig. 3), the tasks Priority Weight, solution scores and the workload balance per worker, provided by the Decision Support module. Except of the visualization of information, this interface provides to the user the ability to load any new data, which will be used by the Decision Support module for the creation of a work schedule or the setup of a simulation run. These data can be either uploaded from a file, or entered manually through the interface. Furthermore, this interface supports the visualization of the shifts work schedule in a timeline graph, and provides several notifications regarding the changes or events occurring on the shop floor. This allows the manager to better handle the schedule of each shift. Using the interactive timeline graph, the manager can also manually reschedule and re-assign tasks to different workers or machines on demand.

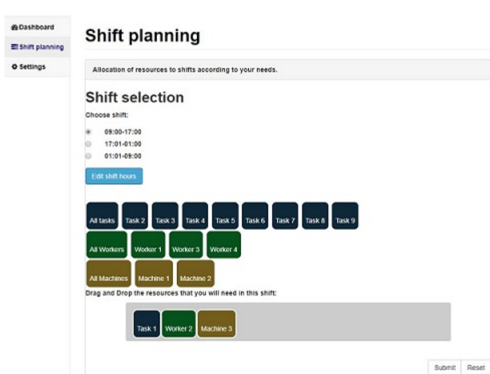

Fig. 3. Assigned tasks per worker and shifts work schedule displayed in the timeline graph.

IV. CASE STUDY AND SIMULATION RESULTS

In order to evaluate the task assignment method and the functionalities of the TDE in general, a case study has been conducted. The case study simulates the insertion of an event occurring on the shop floor on a shift from 09:00 to 17:00. The new event is a new task introduced on the shop floor on a certain time and with certain variables, causing the re-adaptation of the initial work schedule. The aim of the use case is to mainly present the differences on the work-schedule re-adaptation as far as task to human resources allocation concerns, due to different settings in priority levels. The TDE will use tasks, machine and human models in order to produce the initial and re-adapted work schedule of a production line. Regarding each task, variables that denote the complexity of the task and its requirements with regard to workers and machines are utilized. Furthermore, the worker model includes variables describing capabilities, wellbeing status and preferences. Assigning tasks according to worker preferences can result in increased worker satisfaction and can potentially lead to improved productivity. The machine model includes static variables describing the characteristics of the machine, and variables related to its operation (Table I).

TABLE I
VARIABLES USED IN EACH DATA MODEL

M_T	M_R (WORKER)	M_R (MACHINE)
TypeID	Trade	TypeID
Description	Experience level	Description
Estimated duration (minutes)	Well-being rating	Installed location
Priority level (1:Normal, 2:High, 3:Critical)	List of preferred task types	Max allowed operation time (minutes per 24h)
Minimum experience level required	List of preferred working locations	Max allowed consecutive operation time (minutes)
Machine type required		
Interruption property (true, false)		
Complexity (1-100)		
Disturbances probability (1-100)		
Required accuracy level (1-4)		

The initial work schedule concerns the assignment of 9 tasks to 4 workers and 3 machines (Fig. 4). The dataset used for this simulation on behalf of the tasks includes 4 tasks with high priority and minimum requirements of worker expertise, 3 tasks of normal priority and need for novice worker expertise and 2 tasks with normal priority level and experienced worker requirement. Machines 1 and 2 are placed at a location named L1, while machine 3 is located at L2. The dataset of the workers is presented in Table II. The variables of the models are synthetic and not obtained by measurement. The event of a new task includes the characteristics mentioned in Table III. The type of the new task, which will arrive during the simulation, is in the list of tasks preference of Worker with ID 1 only.

Two simulation cases (A, B) were executed, in order to examine and evaluate possible differences on the output. In both simulations the same input parameters were used, except of the settings which define the priority levels of the factors

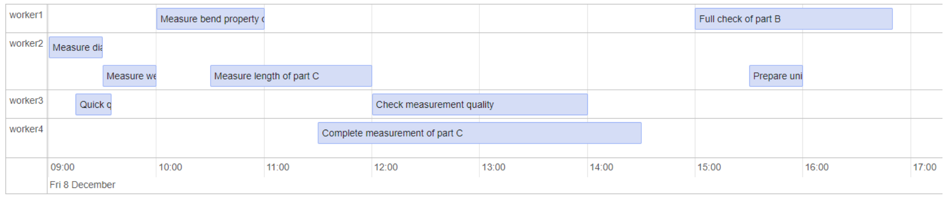

Fig. 4. Initial work schedule of the day containing the scheduled tasks.

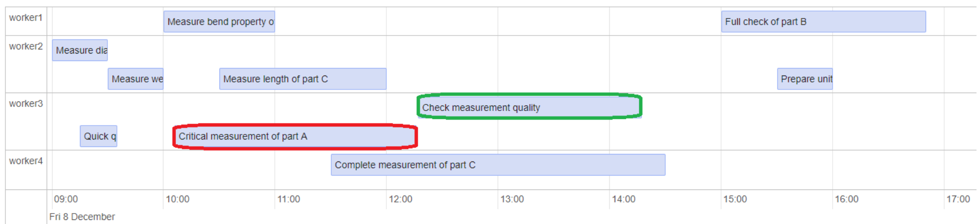

Fig. 5. Work schedule after the arrival of the new task and its assignment to Worker 3 (first simulation run).

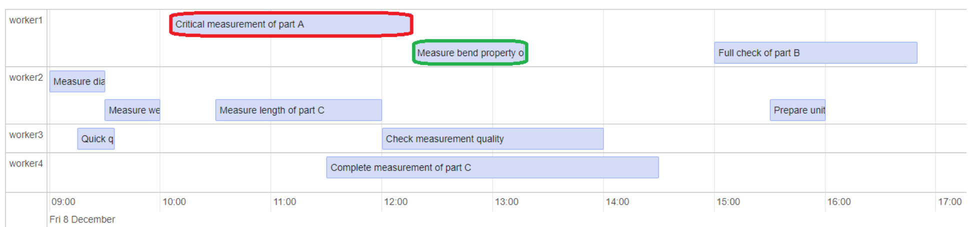

Fig. 6. Work schedule after the arrival of the new task and its assignment to Worker 1 (second simulation run).

TABLE II
DATASET FOR WORKERS INCLUDED IN THE SIMULATION

Worker	Capabilities	Wellbeing rate	Preferences	
			Task type	Location
1	Experienced	2	1,2,3,10	L1
2	Expert	3	1,5,9	L1,L2
3	Experienced	2	4,6	L2
4	Novice	3	7,8	L1

TABLE III
VARIABLES OF THE NEW TASK ENTERED AT THE WORK SCHEDULE

New Task Variables	Values
Description	Critical measurement of part A
Estimated duration (minutes)	130
Priority level	3
Minimum experience level required	Any level of experience
Interruption property	true
Complexity (1-100)	90
Disturbances probability (1-100)	10
Required accuracy level (1-4)	1
Arrival time at simulation	10:05

denoting the soft constraints. Table IV presents the priority levels (PL) of the most important soft constraints in each simulation run. The value of the priority levels reflects also the importance of the corresponding attribute.

TABLE IV
PRIORITY LEVELS OF TDE IN EACH SIMULATION RUN

		Attribute - Soft constraints			
		Work-load	Well-being	Task Prefe-rence	Location Prefe-rence
Simulation	*(A)*	2	2	1	1
	(B)	1	3	2	1

A. Results of simulation run (A)

The re-adapted work schedule after the insertion of the new task was shown to the UI along with a notification during the simulation run, and is illustrated in Fig. 5. It can be observed that the new task (Critical measurement of part A) has been scheduled to start immediately at 10:06 due to its very high TPW, which is equal to 0.95, and it has been assigned to Worker 3. The task named Check measurement quality, which was scheduled to start at 12:00 in the initial work schedule, and is of lower priority, was finally rescheduled to start right after the end of the new task, in order to resolve the overlapping issue. It is worthwhile to note that the new task has been assigned to machine with ID 2, which is located at L2. The workload assigned to each worker after the insertion of the new task is depicted in the UIs respective graph (Fig. 7(a)).

B. *Results of simulation run (B)*

In the second simulation run, the user has entered different priority levels for each attribute, as shown in Table IV. The re-adapted work schedule after the insertion of the new task is depicted in Fig. 6. In this case, the new task has been scheduled to start immediately as well, but it has been assigned to Worker 1 instead, due to the different priorities set. Worker 1 was performing the task Measure bend property of part D, scheduled to start at 10:00, when the new task arrived. The particular task is interruptible, therefore, it was interrupted and scheduled to be resumed after the end of the new task. In this case the distribution of workload among the workers is somewhat more unbalanced, as workload was considered less important according to the users preferences (Fig. 7(b)). Table V presents the scores computed per each assignment solution in order to select the most appropriate worker, for both simulation runs performed.

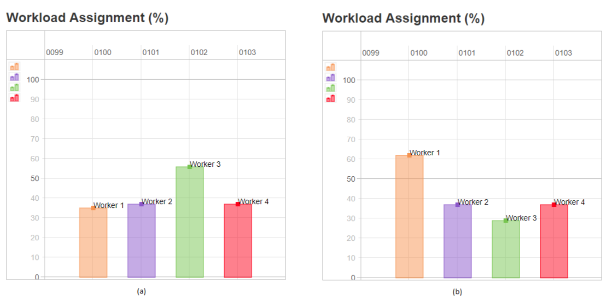

Fig. 7. Workload assigned per each worker after the (a) first simulation run and (b) second simulation run.

TABLE V
SCORE OF THE UTILITY FUNCTION PER EACH SOLUTION

WorkerID		1	2	3	4
Simulation	*(A)*	2.707	2.420	2.919	2.295
	(B)	3.353	1.710	2.459	2.313

V. CONCLUSION

This paper has presented a Task Distribution Engine, which supports both real-time, and simulation operation. The latter, which is the primary topic of interest in this work, allows the evaluation of different scenarios that can occur on the shop floor regarding the assignment and execution of tasks. Static and dynamic variables included in the data models are considered by the system in order to make decisions towards solving the resource assignment problem. In particular, the system is able to assign human and machinery resources by taking into account multiple criteria and re-adapt the work schedule accordingly after the occurrence of certain events. Automated task prioritization is supported in order to set the execution order and starting time of tasks. Furthermore, a constrained optimization method is applied, which considers both hard and soft constraints, in order to insert a new task to the work schedule and assign the most appropriate resource. The Task Distribution Engine employs a Web-based visualization interface, which allows the manager to properly configure the system and conveniently review the output. The system supports excessive features, and includes characteristics of the state-of-the-art Decision Support Systems existing in the literature. Even though the TDE presented is more focused on industrial environments, it could be also applied to other resource assignment domains easily.

Simulation results of the presented case study, demonstrate the impact of the priority levels set by the supervisor related to the different aspects considered, on the resulting work schedule in the case of the arrival of a new task. From the results presented, arises that with the change of the basic parameters in the Decision Support module, such as the priority levels, the work schedule changes accordingly in order to meet the constraints and requirements of each case.

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