3D Content-based Search Based on 3D Krawtchouk Moments

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Abstract

In this paper a novel method for 3D content-based search and retrieval is proposed. Guided by the imperative need for a reliable 3D content based search tool and the very interesting results of research work done in the past on the performance of Krawtchouk moments and Krawtchouk moment invariants in image processing, Weighted 3D Krawtchouk moments are introduced for efficient 3D analysis which are suitable for content-based search and retrieval applications. The proposed method was tested on Princeton Shape Benchmark. Experiments have shown that the proposed method is superior in terms of precision-recall comparing with other well-known methods reported in the literature.

1. Introduction

3D shape matching has evolved to a wide research area during the last years. At the same time, a variety of emerging applications, such as CAD and games design, computer animations, manufacturing and molecular biology applications, dictates the need for efficient 3D search and retrieval tools. Among the several approaches introduced for 3D shape matching, the most well-known ones are based on low-level geometrical characteristics, which can be effectively extracted from the global shape of a 3D object.

The efficient and simple query-by-content approach has been almost universally adopted in the literature, until now. Any such method, however, must first deal with the proper positioning and orientation of the 3D models. The two common methods for the solution to this problem are the pose normalization, where models are placed in a normalized coordinate frame, and native descriptor invariance, where the models are described in a transformation invariant manner. Most of the existing methods for 3D content based search and retrieval, are utilizing the pose normalization method.

2. Previous Work

Many methods for 3D shape search and retrieval have been presented in the literature. In [1], Kolonias *et al.* present a fast query by example approach where the descriptors are properly chosen in order to follow the basic geometric criteria which humans usually use for the same purpose, as the aspect ratio, the angles and the edges of critical points, while Ohbuchi *et al.* [2] present a method based on shape histograms. Three shapes histograms, each one on each principle axis, are discretely parameterized and used to measure the shape similarity.

Daras *et al.* [5], propose a 3D search and retrieval method based on Generalized Radon Transform(GRT). Bustos *et al.* [6] focus on improving the effectiveness of similarity search in 3D object repositories from a system-oriented perspective and propose a heuristic selection, called purity, for choosing retrieval methods based on query-dependent characteristics. Zaharia [7] introduces a new shape descriptor, the Canonical 3D Hough Transform Descriptor which is topologically stable, but not invariant to geometric transformations.

In [8] topology matching is proposed as an interesting and intricate technique. However, the small-scale application field of the method makes it unable to face up general purpose 3D model databases. The MPEG group has proposed [9] the shape spectrum descriptor, which is defined as the histogram of the shape index, calculated over the 3D object's surface. Novotni and Klein [12] exploit and extend 3D Zernike moments introduced by Canterakis [13], for 3D searching. Zernike moments are based on Zernike polynomials and are affine invariants inside the unitary sphere. Suzuki [14] proposes a 3D shape descriptor which is invariant under 90 degrees rotations around coordinate axis.

Funkhouser *et al.* [10] developed a web-based 3D search and retrieval system. The system is capable of indexing large repository of computer graphics models, querying based on text keywords, 3D sketches, 2D sketches and 3D models. The matching algorithm utilizes the spherical harmonics to compute similarities, without a pose normalization to be needed. In [11] Kazhdan et al. present a tool for transforming rotation dependent spherical and voxel shape descriptors into rotation invariant ones. The main idea of this approach is to describe a spherical function in terms of the amount of energy it contains at different frequencies. The results indicate that spherical harmonic representation improves the performance of most of the descriptors.

In [15], Vranic considers 3D-shape descriptors generated by using functions on a sphere. The descriptors are engaged for retrieving polygonal mesh models. Rotation invariance of descriptors achieved with PCA or by defining features in which the invariance exists. A new rotation invariant feature vector based on functions on concentric spheres, that outperformed all recently proposed descriptors is defined and two approaches for achieving rotation invariance as well as options to use a single function or several functions on concentric spheres to generate feature vectors are compared.

In [16], Chen *et al.* propose a visual similarity-based 3D model retrieval system. The main idea is that if two 3Dmodels are similar, they also look similar from all viewing angles and a hundred orthogonal projections of every object are encoded using Zernike moments and Fourier descriptors. The visual similarity-based approach is robust against similarity transformation, noise and model degeneracy, and provided better performance in terms of precision-recall diagram than many other approaches.

The major drawback of the native invariant methods (those methods that does not ?appose? the normalization step) is a loss in discriminative power. For example, spherical harmonics and 3D Zernike moments achieve rotation invariance by computing the Euclidean norm of descriptor vectors, which results in loss in discriminative power. On the other hand, methods utilizing the pose normalization step, can generally result in a description of the object that contains highly discriminative information. However, most of the proposed methods, require either high preprocessing or process time, or very large amount of memory and stored data. In addition, there is no normalization method robust for 3D content-based search and retrieval. Two methods have been proposed for pose normalization, the Principal Component Analysis (PCA) and an affine normalization method proposed by Canterakis [13]. However, the mathematical structure of the latter method suffers from a per-axis scaling step which produces misshaped objects, unsuitable for 3D content-based search applications.

In this paper, a novel compact method suitable for efficient 3D content-based search, is proposed. Guided by the very interesting results of Krawtchouk moments in image processing [20], the Weighted 3D Krawtchouk moments are introduced. Given a 3D object as input, the Weighted 3D Krawtchouk moments are computed, which are then used as a descriptor vector. In this way, a very compact description of a 3D object in the form of a highly discriminative descriptor vector is achieved. The descriptor extraction is very fast and the matching process, one-to-all, for a single object in a medium size database can be completed in few seconds. The method is not invariant under geometrical transformation, thus for every query 3D model a preprocessing pose and position normalization step is required. However, in this paper, the assumption that the rotation and orientation problems are solved has been made. The advantages of Weighted 3D Krawtchouk moments for 3D model analysis derived from their definition. Weighted 3D Krawtchouk moments are based on Weighted Krawtchouk polynomials which are defined on the discrete field; hence, no error is inserted during the moment computation due to discretization. In addition, lower order Weighted Krawtchouk polynomials have relatively high spatial frequency components. Therefore, the Weighted 3D Krawtchouk moments have the ability to represent edges more effectively with lower order moments and the computed descriptor vectors have the ability to capture more information retaining a low dimensionality and thus producing better retrieval results.

The paper is organized as follows. In Section 3 the Weighted 3D Krawtchouk Moments are introduced in terms of Weighted Krawtchouk polynomials. Computational aspects of Krawtchouk Moments are presented in Section 4 while in Section 5 the matching method is described. In Section 6 the experimental results evaluating the proposed method and comparing it with other methods are presented. Finally conclusions are drawn in Section 7.

3. Extraction Of Krawtchouk Descriptors

In this section, the mathematical background needed for the introduction of Weighted 3D Krawtchouk moments is presented. Then, the Weighted 3D Krawtchouk moments are introduced. The background of Krawtchouk polynomials presented below can be also found in [20].

3.1. Simple Krawtchouk Polynomials

Krawtchouk moments are based on a set of orthonormal polynomials, associated with the binomial distribution, introduced by Mikhail Krawtchouk almost 80 years ago. More recent approaches expressed Krawtchouk polynomials in terms of hypergeometric function [17, 18].

The *n*-order Krawtchouk classical polynomials [19] are defined in terms of hyper-geometric function as:

$$K_n(x;p,N) = \sum_{k=0}^{N} a_{k,n,p} x^k = {}_2F_1(-n,-x;-N;\frac{1}{p}) \tag{1}$$

where $x, n = 0, 1, 2 \ldots N, N > 0, p \in (0, 1)$ and the function ${}_2F_1$ is the hypergeometric function which is defined as:

$$ {}_2F_1(a,b;c;z) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{(c)_k} \frac{z^k}{k!} \quad (2) $$

The symbol $(a)_k$ in (2) is the Pochhammer symbol given by

$$ (a)_k = a(a+1)(a+2)\dots(a+k-1) = \frac{\Gamma(a+k)}{\Gamma(a)} \quad (3) $$

The set of Krawtchouk polynomials $S = \{ K_n(x;p,N), n = 0 \dots N \}$ has $N+1$ elements and forms a complete set of discrete basis functions with weight function

$$ w(x;p,N) = \binom{N}{x} p^x (1-p)^{N-x} \quad (4) $$

Using the properties of hyper-geometric function, it can be proved that:

$$ \sum_{x=0}^{N} w(x;p,N) K_n(x;p,N) K_m(x;p,N) = \rho(n;p,N)\delta_{nm} \quad (5) $$

where $n, m = 1, 2, 3 \dots N$,

$$ \rho(n;p,N) = (-1)^n \left(\frac{1-p}{p} \right)^n \frac{n!}{(-N)_n} \quad (6) $$

and δ_{nm} is the Kronecker delta function.

The Equation (5) shows that the set S satisfies the orthogonality condition.

3.2. Normalized and Weighted Krawtchouk Polynomials

As mentioned, the set S of Krawtchouk polynomials is orthogonal, however, it is not orthonormal. In order to achieve orthonormality, the normalized Krawtchouk polynomials are defined as:

$$ \tilde{K}(x;p,N) = \frac{K_n(x;p,N)}{\sqrt{\rho(n;p,N)}} \quad (7) $$

Using (5) can be shown that

$$ \sum_{x=0}^{N} \sum_{y=0}^{M} w(x;p,N) \tilde{K}_n(x;p,N) \tilde{K}_m(x;p,N) = \delta_{nm} \quad (8) $$

In order to ensure the numerical stability of the polynomials and to achieve an orthonormal basis function with unitary weight function, Yap et al [20] introduced the set of weighted Krawtchouk polynomials, defined as:

$$ \bar{K}(x;p,N) = K_n(x;p,N) \sqrt{\frac{w(x;p,N)}{\rho(n;p,N)}} \quad (9) $$

Therefore the orthogonality condition (5) becomes

$$ \sum_{x=0}^{N} \sum_{y=0}^{M} \bar{K}_n(x;p,N) \bar{K}_m(x;p,N) = \delta_{nm} \quad (10) $$

Thus, normalized and weighted Krawtchouk polynomials can be used as orthonormal function basis of discrete space $[0 \dots N-1]$

3.3. Weighted 3D Krawtchouk Moments

In [20], Yap *et al.* introduced Krawtchouk moments and Krawtchouk moment invariants for image analysis, 2D object recognition and region based feature extraction (2D case). In this section, this work is extended in 3D and the discrete Weighted 3D Krawtchouk moments are introduced.

Let $f(x,y,z)$ be a 3D function defined in a discrete field $A = \{(x,y,z) : x,y,z \in I\!N,\ x = [0 \dots N-1],\ y = [0 \dots M-1],\ z = [0 \dots L-1]\}$. Exploiting Weighted Krawtchouk polynomials, the Weighted 3D Krawtchouk moments are introduced. Weighted 3D Krawtchouk moments of order (n+m+l) of f, are introduced as follows:

$$ \begin{aligned} \bar{Q}_{nml} \quad = \quad & \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} \sum_{z=0}^{L-1} \bar{K}_n(x;p_x,N-1) \times \\ & \times \bar{K}_m(y;p_y,M-1) \bar{K}_l(z;p_z,L-1) \times \\ & \times f(x,y,z) \end{aligned} \quad (11) $$

By solving the orthogonality condition (10) and the definition of Weighted 3D Krawtchouk moments (11), function $f(x,y,z)$ can be written in terms of Weighted 3D Krawtchouk Moments as:

$$ \begin{aligned} f(x,y,z) \quad = \quad & \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} \sum_{l=0}^{L-1} \bar{K}_n(x;p_x,N-1) \times \\ & \times \bar{K}_m(y;p_y,M-1) \bar{K}_l(z;p_z,L-1) \times \\ & \times \bar{Q}_{nml} \end{aligned} \quad (12) $$

Equations (11) and (12) show that any 3D function $f(x,y,z)$ defined in a discrete 3D field, can be decomposed into the appropriate Weighted 3D Krawtchouk representation $\tilde{Q}_{nml}$. Moreover, the projection of f to the space of Weighted 3D Krawtchouk moments is fully reversible.

3.4. Application of Weighted 3D Krawtchouk Moments for 3D objects

Weighted 3D Krawtchouk moments can be used as a descriptor of any 3D object, if it can be expressed as a function $f(x,y,z)$ defined in a discrete space $[0 \dots N-1] \times [0 \dots M-1] \times [0 \dots L-1]$. This can be achieved if the model is expressed as a binary volumetric function.

However, a 3D model M is generally described by a 3D mesh. In order to compute the Weighted 3D Krawtchouk moments, the 3D mesh representation has to be converted into a volumetric representation of the 3D object. This can be achieved by utilizing an appropriate voxelization method. A brief overview over the principles of the voxelization method used in this paper is presented below :

Let $N \times N \times N$ be the size of cube with axes parallel to the system coordinates and bounding the mesh. The bounding cube is partitioned in R^3 equal cube shaped voxels $\boldsymbol{u}_i$ with centers $\boldsymbol{v}_i$. The size of each voxel is $(\frac{N}{R})^3$. Let $\boldsymbol{U}$ be the set of all voxels inside the bounding cube and $\boldsymbol{U}_M \subseteq \boldsymbol{U}$, be the set of all voxels belonging to the bounding cube and lying inside M. Then, the discrete binary volume function $\hat{f}(\boldsymbol{v})$ of M, is defined as:

$$\hat{f}(\boldsymbol{v}) = \begin{cases} 1, & \text{when } \boldsymbol{v} \in \boldsymbol{U}_M, \\ 0, & \text{otherwise.} \end{cases}$$

In general, 3D models have various levels-of-detail, depending on their mesh representation, ranging from a very few to thousands of vertices and triangles. The number R^3 of voxels is kept constant for all models in order to achieve robustness with respect to the level-of-detail [21].

After normalization and generation of a binary volumetric function of the 3D object, the Weighted 3D Krawtchouk moments can be computed. These Weighted 3D Krawtchouk moments can then be used to form the descriptor vector of every object. Specifically, the descriptor vector is composed of Weighted 3D Krawtchouk Moments up to order s, where s is experimentally selected.

$$D = \left[\bar{Q}_{nml} | n+m+l \in [0 \dots s]\right] \tag{13}$$

4. Computational Aspects on Krawtchouk Moments

It can be easily seen that a brute force implementation of Weighted 3D Krawtchouk Moments is a very heavy computational task, as the complexity reaches $O(n^6)$. To overcome this obstacle, the following recurrent relations are used [19]:

$$\begin{aligned} K_{n+1}(x,p,N) &= \left(1 + \frac{n - np - x}{pN - pn}\right) K_n(x;p,N) - \\ &- \frac{n - np}{pN - pn} K_{n-1}(x;p,N) \end{aligned} \tag{14}$$

$$w(x+1;p,N) = \frac{w(x;p,N)p(N-x)}{x+1-p-xp} \tag{15}$$

The initial conditions are:

$$K_0(x;p,N) = 1 \tag{16}$$

$$K_1(x;p,N) = 1 - \frac{1}{Np}x \tag{17}$$

$$w(0;p,N) = (1-p)^N \tag{18}$$

5. Matching Method

Let A, B, be two 3D models and $D^A = [\bar{Q}^A_{nml}], n+m+l = 0 \dots s], D^B = [\bar{Q}^B_{nml}, n+m+l = 0 \dots s]$, (from (11) and (13)) their Weighted 3D Krawtchouk moments descriptors respectively. The models are compared in pairs in terms of Weighted 3D Krawtchouk moments using the $L1 - norm$ between D^A and D^B:

$$H(A,B) = L1(D^A, D^B) = \sum_{n+m+l=0}^{s} \left|\bar{Q}^A_{nml} - \bar{Q}^B_{nml}\right| \tag{19}$$

where s is the maximum order of Weighted Krawtchouk Moments selected to describe the object. The matching is performed by comparing the query model against the models in the database and increasingly ranking the computed distances.

6. Experimental Results

The proposed method was tested using the Princeton Shape Benchmark Database [22] for its performance on 3D content-based search and retrieval application. The dataset consists of 907 3D models classified into 35 main categories and 92 subcategories. In this paper, the assumption that the rotation and orientation problems are solved has been made. Thus, a version of pre-rotated Princeton Shape Benchmark Database has been used and the preprocessing step has been omitted.

The Princeton Shape Benchmark consists of 3D models in VRML format. Therefore, a preprocessing step for converting the mesh representation into a volumetric representation was needed in order to compute the Weighted 3D Krawtchouk Moments for each model. The 3D mesh is enclosed in the smallest bounding cube which is then partitioned in a set of equal cube shaped voxels, using the method presented in [5]. The resolution of the bounding box was selected to be $64 \times 64 \times 64$ voxels.

The retrieval performance was evaluated in terms of “precision” and “recall”, where precision is the proportion of the retrieved models that are relevant to the query and recall is the proportion of relevant models in the entire database that are retrieved in the query [5].

The parameters required for moments extraction are selected to be $p_x = p_y = p_z = 0.5$, because the mass center of the object lies at the center of the voxel model and $N = M = L = 64$ because of the voxel model dimensions. Weighted 3D Krawtchouk moments for all objects

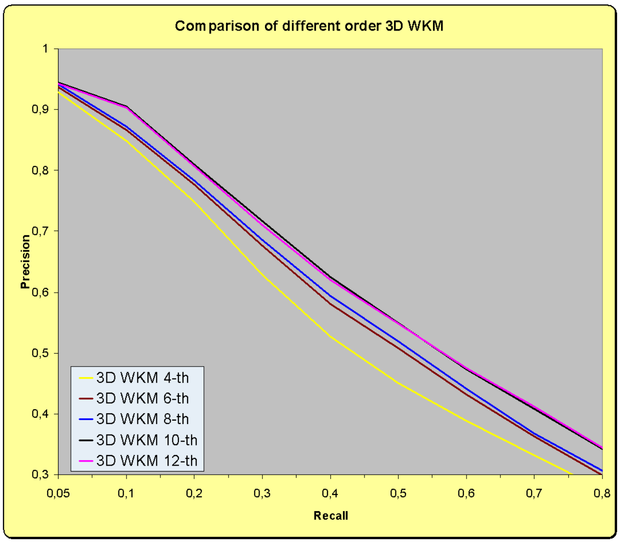

Figure 1: Comparison of different order moments the proposed method (**<u>W</u>eighted <u>K</u>rawtchouk <u>M</u>oments**).

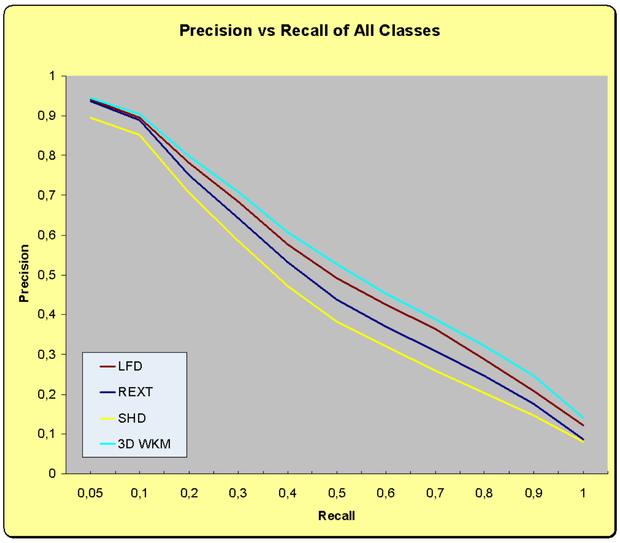

Figure 2: Comparison of the proposed method (**<u>W</u>eighted <u>K</u>rawtchouk <u>M</u>oments**) against the methods REXT, Light Field Descriptor (LFD) and Spherical Harmonics (SHD) proposed in **[15]**, **[16]** and **[11]** respectively, in terms of precision-recall diagram, using the Princeton Shape Benchmark.

have been computed for $n + m + l \in [0 \dots s]$. The value of s has been experimentally selected to be $s = 10$ because produces better and more stable results. For $s > 10$, the number of descriptors increases significantly without considerable improvement in retrieval performance. Figure 1 presents the results produced for different values of $s = 4, 6, 8, 10, 12$ in terms of precision-recall on the Princeton Shape Benchmark.

To evaluate the ability of the proposed method to discriminate between classes of objects, each 3D model was used as a query object. Our results (3D WKM) were compared with those of the methods of Spherical Harmonics (SHD) [11], Light Field Descriptor (LFD) [16] and REXT method [15] which are some of the best known shape matching methods. The resulting precision-recall diagram is presented in Figure 2.

It has to be noted that we did not implement the above methods. All executables were taken from the home pages of the authors of [11, 16, 15].

These results were obtained using a PC with a 3 GHz Pentium IV processor and $512MB$ RAM, running operating system windows 2000. The programs have been compiled with Microsoft Visual C++ Compiler version 6. The average time needed for the extraction of the feature vectors for one 3D model is 1.01 seconds, while the time needed for the retrieval process with a single query model is 10 msec. The time needed for the retrieval process depends on the descriptor vector size, thus is constant.

Table 1 presents analytically the times required for descriptor extraction with Weighted 3D Krawtchouk moments of order up to 8, 10 and 12 and the corresponding retrieval times. The Weighted 3D Krawtchouk moments extraction time depends on the total number of voxels which constitute the object and the maximum order of computed moments. Therefore, Weighted 3D Krawtchouk moments for smaller object are computed faster than those for bigger objects. Thus, in Table 1 the minimum, the maximum and the average extraction times are presented. The matching time depends on the size of the database and the size of the descriptor vector. Thus, there is a constant time for the descriptors comparison for Krawtchouk moments of the same order. Furthermore, Table 1 justifies the selection of $s = 10$. Although, for $s = 12$ the results are comparative with $s = 10$, (Figure 1) the time required for descriptor extraction and matching process is increased significantly.

Table 1: Execution Times in Princeton Shape Benchmark 3D WKM

		8th	10th	12th
Extraction of 3D WKM Descriptors	Min	0.4 sec	0.731 sec	1.45 sec
	Max	1.2 sec	1.99 sec	3.92 sec
	Aver.	0.58 sec	1.01 sec	2.12 sec
Comparison	All	1 msec	1.5 msec	3 msec

Clearly, the time needed for the extraction of the feature vectors is short, thus the method is appropriate for applications close to real time, provided that the models are expressed in terms of a binary volumetric function. Since the time needed for the comparison of the feature vectors is small, the proposed method is suitable to be used as an efficient tool for web-based, real-time search and retrieval applications.

Figure 3 illustrates some of the results produced by the proposed method. The first model in each horizontal line is the query model while the rest are the first four retrieved models. The similarity between the query model and the retrieved ones is obvious.

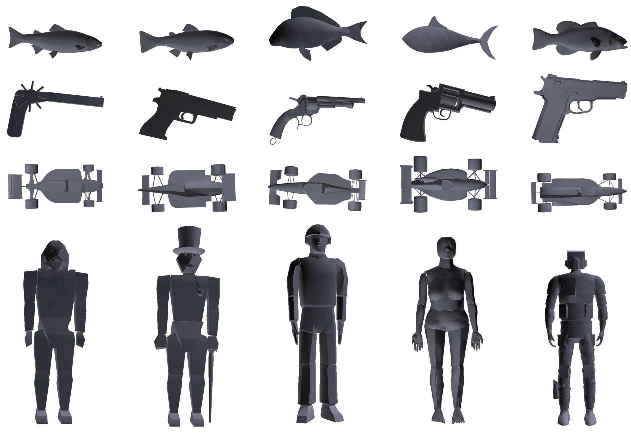

Figure 3: Query results using the proposed method in the Princeton database. The query models are depicted in the first horizontal line.

The efficiency of Weighted 3D Krawtchouk Moments to capture the object edges can be figured in the precision-recall diagram of a specific category in Princeton Shape Benchmark (Figure 4). The diagram compares the results retrieved using Weighted 3D Krawtchouk moments and Spherical Harmonics for a specific category which contains models with many edges. It is obvious that the proposed method produces much better results for this category. Furthermore, Figure 5 comparatively presents the retrieved results of the same query model using the Weighted 3D Krawtchouk Moments (3D WKM) and Spherical Harmonics Descriptors (SHD). The results prove that the proposed method can effectively capture edges with low order moments.

Figure 6 illustrates the retrieved results using the proposed method with an aerostat as query. The results show that the retrieved objects are semantically irrelevant, as the first three retrieved results are not aerostats. However, the shape similarity between the retrieved results is obvious.

7. Conclusions

In conclusion, the Weighted 3D Krawtchouk moments outperforms other known methods due to

- Krawtchouk polynomials and Weighted 3D Krawtchouk moments are defined in the discrete field, while other methods, like Spherical Harmonics, are defined in a continuous field. This is a major advantage of the method, because no discretization error is inserted during the Weighted 3D Krawtchouk moments analysis.

- Krawtchouk polynomials and Weighted 3D Krawtchouk moments of low order have relatively high spatial frequency components. Therefore, Weighted 3D Krawtchouk moments can capture sharp shape changes of the object.

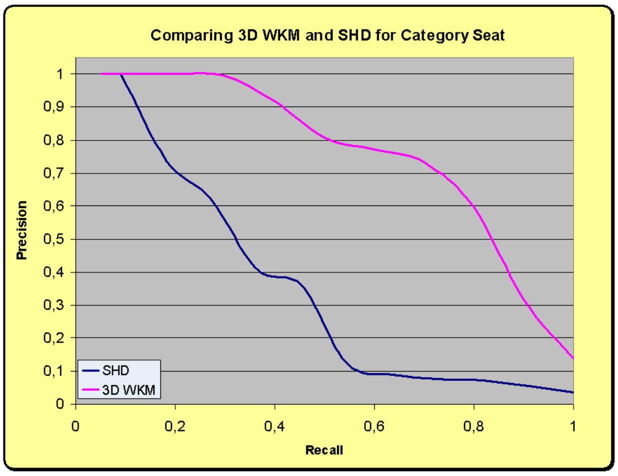

Figure 4: Comparison of the proposed method (**Weighted Krawtchouk Moments**) against the method of Spherical Harmonics (SHD) proposed in **[11]**, in terms of precision-recall diagram, using the Princeton Shape Benchmark.

Figure 5: Retrieved results with 3DWKM (first line) and SHD (second line) for the same query model. The first model is the query model and the rest are the retrieved results.

Figure 6: Retrieved results with 3DWKM for an aerostat. The first model is the query model and the rest are the retrieved results

In this paper a novel approach for 3D content-based search and retrieval was presented. The proposed method is based on the introduced Weighted 3D Krawtchouk moments, which form a very compact and highly discriminative descriptor vector, due to their ability to capture sharp changes in the volumetric function with low order moments. The proposed method was evaluated to Princeton Shape Benchmark and our results outperformed three other well-known methods reported in the literature.

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