

Ellipsoidal Harmonics for 3-D Shape Description and Retrieval

Athanasios Mademlis, Petros Daras, *Member, IEEE*, Dimitrios Tzovaras, and Michael Gerassimos Strintzis, *Fellow, IEEE*

Abstract—In this paper, a novel approach for 3-D Shape description and retrieval based on the theory of ellipsoidal harmonics is presented. Four novel descriptors are introduced: the surface ellipsoidal harmonics descriptor, which concerns 3-D objects that are described as polygonal surfaces; the volumetric ellipsoidal harmonics descriptor, which is applicable to volumetric 3-D objects; the generalized ellipsoidal harmonics descriptor that is applied to any local 3-D object descriptors; and, finally, the combined ellipsoidal-spherical harmonics descriptor, which leads to a compact and powerful descriptor that inherits the advantages of both approaches: the rotation invariance properties of the spherical harmonics and the directional information enclosed in ellipsoidal harmonics. Experimental results performed using well-known 3-D object databases prove the retrieval efficiency of the proposed approach.

Index Terms—3-D object retrieval, ellipsoidal harmonics, shape description.

I. INTRODUCTION

THE 3-D object retrieval is a relatively new and very challenging research field and a major effort of the research community has been devoted to the formulation of accurate and efficient 3-D object search and retrieval algorithms. In the new 3-D era, the growth of the Internet along with the recent progress in computer's graphical units and the development of friendly and easy-to-use 3-D content creation tools have led to the creation of huge repositories with 3-D objects. In this new environment, the demand for efficient tools that can quickly and accurately retrieve the desired 3-D content is emerging.

Manuscript received July 28, 2008; revised August 04, 2009. This work was supported in part by the VICTORY project funded by the European Commission and in part by the PENED project co-funded 75% from European Commission and 25% by the Greek Secretariat for Research and Technology. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Mohan S. Kankanhalli.

A. Mademlis is with the Information Processing Laboratory, Department of Electrical and Computer Engineering, Aristotle University of Thessaloniki, Thessaloniki, Greece (e-mail: mademlis@iti.gr).

P. Daras and D. Tzovaras are with the Informatics and Telematics Institute, Centre for Research and Technology Hellas, Thessaloniki, Greece (e-mail: daras@iti.gr; Dimitrios.tzovaras@iti.gr).

M. G. Strintzis is with the Information Processing Laboratory, Department of Electrical and Computer Engineering, Aristotle University of Thessaloniki, Thessaloniki, Greece, and also with the Informatics and Telematics Institute, Centre for Research and Technology Hellas, Thessaloniki, Greece (e-mail: strintzi@eng.auth.gr).

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Digital Object Identifier 10.1109/TMM.2009.2032690

A. Related Work

In the last few years, a lot of work has been performed in the area of accurate 3-D shape description for search and retrieval applications. The presented approaches can be classified into four major categories: global feature-based approaches, local feature-based approaches, topology-based approaches, and view-based approaches.

The global feature-based approaches, which are the first methods that appear in the field of 3-D search and retrieval, aim to capture the geometry of the whole object in a single representation (usually a vector), using various methods: primitive shape features [1], [2], moments (Krawtchouk [3], Zernike [4]), transformations, etc. Local feature-based approaches attempt to describe the complete 3-D object using local features. The majority of the local feature-based approaches describes the complete geometry using histograms; however, few attempts to further exploit the local information have been recently presented.

In topology-based approaches, the main feature of the 3-D object is formed by its topology instead of its geometry. The topology is usually represented in the form of a graph, and the matching is performed using dedicated graph matching techniques. The view-based approaches cannot be classified as native 3-D description methods. The object is decomposed to a collection of 2-D views and the most powerful image descriptors are utilized. Table I summarizes some important 3-D object retrieval approaches. For more sophisticated analysis, the reader is referred to the 3-D search and retrieval reviews [5], [6]. More recently, the field of 3-D shape search and retrieval has attracted more researchers and the competition on the field has been increased significantly, mainly due to the Shape Retrieval Contest (SHREC), organized each year by the consortium of Aim@Shape EU-funded project [7].

All of the existing approaches present advantages and drawbacks. Topology-based approaches are the only approaches that can capture topological information; however, few methods exist that equally rely on both topology and geometry. Additionally, the majority of the topology-based approaches cannot easily generalize to all kinds of 3-D objects and they are very sensitive to minor shape changes (the topology can be seriously altered); thus, their applicability is limited to few classes of 3-D objects. Local feature-based approaches focus on acquiring highly discriminant local representations, which are usually integrated in a single (or multiple) histogram(s) and thus, their discriminative power is seriously affected in the majority of the approaches. View-based approaches provide very reliable 3-D shape representation; however, due to their nature, they

TABLE I
RELATED WORK PRESENTED IN THE LITERATURE

Global Feature	Primitive shape features: Volume, area, moments [1], bounding box, cords, moments, wavelets [14], convex hull features (crumpliness, packing and compactness) [2], Spatial Maps of volumetric features [15], [10] Transformations: Generalized Radon Transform [16], Spherical Harmonics [9], 3D Zernike [17], complex spherical functions [18], [19], 3D angular radial transform (ART) [20]
Local Feature	Forming Histograms: MPEG-7 shape index [21], Extended Gaussian Images [22], Complex Extended Gaussian Images [23], Space partitioning histograms [10] Spin Image Singatures [24], Priority-Driven Search [25].
Topology Based	Curve Skeletons [26], [27], Reeb Graphs[28], Medial Surfaces [29]
View Based	Light-Field Descriptor [11]

TABLE II
COMPARISON OF RELATED WORK PRESENTED IN THE LITERATURE

Method	Invariance				Generality
	Rotation	Translation	Scale	Articulation	
[16]	PCA	Normalization	Yes	No	Yes
[9]	Yes	Normalization	Normalization	No	Yes
[17]	Yes	Normalization	Normalization	No	Yes
[20]	PCA	Normalization	Normalization	No	Yes
[25]	Yes	Yes	Yes	Yes	Yes
[28]	Yes	Yes	Yes	Yes	Yes
[29]	Yes	Yes	Yes	Yes	No
[26]	Yes	Yes	Yes	Yes	No
[11]	Search	Normalization	Normalization	No	Yes

Legend:

PCA: Rotation Invariance is achieved using Principal Component Analysis. Normalization: A normalization step is required before applying the method. For scale and translation normalization is robust. Search: the invariance is achieved during search using complex techniques.

are unable to capture features that cannot be seen from the selected points of view. Also, their time performance during retrieval can be considered as a serious disadvantage (N to N matching is required). Global feature-based approaches are more reliable when compared to topology-based and local feature-based approaches due to their ability to capture mainly the global geometry of the 3-D object discarding local features, i.e., minor shape changes between two objects do not seriously affect the global shape description. The latter is suitable for the general purpose 3-D object retrieval applications where the retrieval is based on global shape similarity. However, it can also be considered as a potential drawback in some cases, when there are objects of the same class that can dramatically change their shape (non-rigid objects or, usually, articulated objects), or objects from different classes that present major similarities (e.g., spindle of the helicopters has similar geometry to a fish).

There are also other problems that pose obstacles in the efficiency of the existing global-based approaches, such as the 3-D object's degeneracies (e.g., holes, missing polygons, hidden polygons), the 3-D object's pose normalization, the retrieval accuracy, etc. The first problem is usually tackled successfully by applying a triangulation algorithm (e.g., Delaunay triangulation) or a hole filling algorithm [8]. Concerning the pose normalization problem, there are two widely acceptable solutions presented in the literature: the natively rotation invariant description of the 3-D object (e.g., using spherical harmonics [9] histogram-based descriptors [10]) or natively rotation invariant matching (e.g., light field descriptor [11]) and the rotation normalization of the object in a preprocessing step. Both approaches present major advantages and serious drawbacks: Firstly, the vast majority of the utilized rotation normalization approaches are based on the PCA (e.g., continuous PCA [12]).

Although algorithms that utilize pose normalization using PCA usually result in descriptors with higher discriminative power, some similar objects are not usually normalized in a similar manner [13]. In contrast, natively rotation invariant object description [9] usually involves an integration-like technique which leads to descriptors which are not adequately discriminant [12]. Table II summarizes the properties of some approaches presented in the literature.

B. Motivation and Contributions of the Proposed Work

As it was earlier mentioned, one of the major problems of the global approaches is the trade-off between rotation invariance and highly discriminative shape information. In this paper, new geometric descriptors are proposed, which are based on the ellipsoidal harmonics. Ellipsoidal harmonics offer a compact and discriminative object representation that is appropriate for 3-D content-based search and retrieval. The proposed approach can be utilized using both surface-based and volumetric-based 3-D object representation and is invariant under scaling and translation of the 3-D object, using relative distances to the parameters of the bounding ellipsoid. For rotation normalization, an appropriate normalization approach is introduced (without using the well-known traditional PCA). Then, ellipsoidal harmonics analysis is extended and applied to local 3-D descriptors, leading to the generalized ellipsoidal harmonics descriptor. Finally, the directional information of the ellipsoidal harmonic descriptors is combined with spherical harmonics in order to produce a natively rotation invariant descriptor that inherits the properties of both descriptors.

The major contributions of the proposed approach are the following.

- Compact representation: The resulting descriptor set is very compact, i.e., the descriptor vector dimensionality is small.
- Better object approximation: The approximation of a 3-D object using ellipsoids is better than using spheres.
- The method is applicable on both volumetric and surface-expressed 3-D objects.
- The method can easily be generalized in order to utilize any local descriptor.
- The method can easily be combined with spherical harmonics in order to produce natively rotation invariant descriptors.
- The proposed methods are not sensitive to minor shape changes.

The rest of this paper is organized as follows: In Section II, the theory of ellipsoidal harmonics is briefly reviewed. In Section III, the various instances of ellipsoidal harmonics descriptors are described and in Section IV, ellipsoidal harmonics are combined with spherical harmonics. The utilized matching method is presented in Section V, and the experimental results are given in Section VII. Finally, the conclusions are drawn in Section VIII.

II. ELLIPSOIDAL HARMONICS

The ellipsoidal harmonics are special functions that have been utilized in the field of astronomy [30] for describing surrounding force-fields of non-spherical objects. In this paper, the theory of ellipsoidal harmonics is adapted and applied to the field of 3-D shape analysis and description. The basic motivation behind the selection of ellipsoidal harmonics for 3-D object description relies on the intuition that an ellipsoid forms a better approximation of the shape of a 3-D object when compared to the approximation using spheres. The theoretical problem of the ellipsoidal harmonics has been targeted by mathematicians for many years in the early 1900s. Thus, many variations in the notation of ellipsoidal harmonics can be found in the literature (e.g., [31], [32]). For the needs of this paper, the notation utilized in [31] has been adopted, due to its simplicity. For the sake of completeness, the theory of ellipsoidal harmonics is briefly reviewed.

A. Ellipsoidal Coordinates

Intuitively, one of the basis functions in the space of ellipsoidal coordinates has to be an ellipsoid. The latter is verified by the definition of the ellipsoidal coordinates, which are defined using a reference ellipsoid with axes length a, b, c :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1. \quad (1)$$

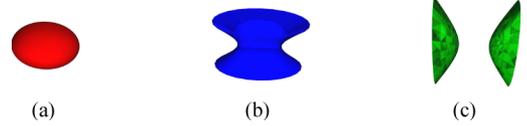


Fig. 1. Ellipsoid $\lambda_1^2 = const$ (a), the one-sheet hyperboloid (b) $\lambda_2^2 = const$, and the two-sheet hyperboloid (c) $\lambda_3^2 = const$.

Without any loss of the generality of the approach, it is assumed that $a \geq b \geq c$. By setting $\lambda_i^2 = a^2$, $h^2 = a^2 - b^2$, and $k^2 = a^2 - c^2$, the above equation can be transformed to

$$\frac{x^2}{\lambda_i^2} + \frac{y^2}{\lambda_i^2 - h^2} + \frac{z^2}{\lambda_i^2 - k^2} = 1. \quad (2)$$

For any given point in the Euclidean 3-D space $P(x, y, z)$, (2) with respect to λ_i^2 , has three discrete solutions, $\lambda_1^2 \in [k^2, +\infty)$, $\lambda_2^2 \in [h^2, k^2]$, and $\lambda_3^2 \in [0, h^2]$, which are called ellipsoidal coordinates. The ellipsoidal coordinates form an orthogonal basis for a curved space that is created by homofocal ellipsoids. For a given (h, k) , the family of ellipsoids, obtained for different values of λ_i^2 , are homofocal. The equations $\lambda_1^2 = const$, $\lambda_2^2 = const$, and $\lambda_3^2 = const$ define an ellipsoid, a hyperboloid of one sheet, and a hyperboloid of two sheets, respectively (Fig. 1). Although the mapping between ellipsoidal and Cartesian coordinates is not one-to-one (because of the squares), this problem can easily be avoided (further information is provided in Section III-D).

B. Lamé Polynomials

Ellipsoidal harmonics are the solutions of the Laplace equation in ellipsoidal coordinates $\nabla^2 V = 0$. The main advantages of this approach (from the mathematical scope of the problem) is spotted in the fact that the solutions of Laplace's equation are orthogonal and separable, i.e., see equation (3) and (4) at the bottom of the page, where $\delta(\cdot)$ is the Kronecker δ function. The Laplace equation in the space of ellipsoidal coordinates is simplified to [30]

$$\begin{aligned} & (\lambda_i^2 - h^2)(\lambda_i^2 - k^2) \frac{d^2 E_n(\lambda_i)}{d\lambda_i^2} \\ & + \lambda_i(2\lambda_i^2 - h^2 - k^2) \frac{dE_n(\lambda_i)}{d\lambda_i} \\ & + (p - n(n+1)\lambda_i^2) E_n(\lambda_i) = 0. \end{aligned} \quad (5)$$

For every n , (5) has exactly $2n$ solutions, the polynomials $E_n^p(\cdot)$ (are known both as ellipsoidal harmonics and lamé polynomials), where $n = 0 \dots \infty$ and $p = 0 \dots 2n$. $E_n^p(\cdot)$ form a complete set of basis functions of the curved space of ellipsoidal

$$\mathbf{E}_n^p(\lambda_1, \lambda_2, \lambda_3) = E_n^p(\lambda_1) E_n^p(\lambda_2) E_n^p(\lambda_3) \quad (3)$$

$$\int \int E_n^p(\lambda_2) E_m^q(\lambda_2) E_n^p(\lambda_3) E_m^q(\lambda_3) dS = \delta(m - n, p - q) \quad (4)$$

coordinates. The n th-degree lame polynomials are n th-degree polynomials of λ^2 and can be classified in four families according to (6), where $r = \lfloor n/2 \rfloor$. There are $r + 1$ polynomials that belong to the family K , $n - r$ polynomials that belong to the families L and M , and r polynomials that belong to family N . See (6) at the bottom of the page.

Due to the polynomial nature of lame polynomials (6) and the values of λ_1, λ_2 and λ_3 , the values of a_j are rapidly vanishing as j increases. Although, theoretically, this is not a major issue, in practice, the limited accuracy of existing computational systems results in inaccurate values of a_j for $j > A$, where A depends on the arithmetic precision utilized. In order to prevent the latter, the above (6) are usually transformed in the form (7) [30]. See (7) at the bottom of the page.

In (7), the equations involve polynomials of $f = (1 - \lambda^2/h^2)$, which is valued $0 \leq f \leq 1$ for $\lambda = \lambda_3$ and $1 - k^2/h^2 \leq f \leq 0$ for $\lambda = \lambda_2$. Using this computation scheme, the values of parameters b_j are not quickly vanishing, allowing more accurate results for higher order polynomials.

Both parameters a_j and b_j for every ellipsoidal harmonic degree are easily computed using appropriate eigenanalysis. Detailed computational issues of ellipsoidal harmonics are above the scope of this paper and the reader is referred to [30] and [31].

C. Ellipsoidal Harmonics Expansion

Taking advantage of the lame's polynomials orthogonality, every 3-D function defined in the space of ellipsoidal coordi-

nates $O(\lambda_1, \lambda_2, \lambda_3)$ can be expressed as a weighted sum of the lame polynomials, i.e.,

$$O(\lambda_1, \lambda_2, \lambda_3) = \sum_{n=0}^{\infty} \sum_{p=0}^{2n} o_n^p E_n^p(\lambda_1) E_n^p(\lambda_2) E_n^p(\lambda_3) \quad (8)$$

where

$$o_n^p = \int \int_S O(\lambda_1, \lambda_2, \lambda_3) E_n^p(\lambda_2) E_n^p(\lambda_3) dS. \quad (9)$$

The o_n^p values can fully characterize every function defined in the space of ellipsoidal coordinates for $\lambda_1 \leq \alpha$. For $\lambda_1 > \alpha$, (8) is modified as

$$O(\lambda_1, \lambda_2, \lambda_3) = \sum_{n=0}^{\infty} \sum_{p=0}^{2n} o_n^p I_n^p(\lambda_1) E_n^p(\lambda_1) E_n^p(\lambda_2) E_n^p(\lambda_3) \quad (10)$$

where

$$I_n^p(\lambda) = \int_{\lambda}^{\infty} \frac{dt}{E_n^p(t) \sqrt{(t^2 - h^2)(t^2 - k^2)}}. \quad (11)$$

$$E_n^p(\lambda) = \begin{cases} K(\lambda) = \lambda^{n-2r} \sum_{j=0}^r a_j \lambda^{2j} & p = [0, r] \\ L(\lambda) = \lambda^{1-n+2r} \sqrt{|\lambda^2 - h^2|} \sum_{j=0}^{r-1} a_j \lambda^{2j} & p = [r+1, n] \\ M(\lambda) = \lambda^{1-n+2r} \sqrt{|\lambda^2 - k^2|} \sum_{j=0}^{r-1} a_j \lambda^{2j} & p = [n+1, 2n-r] \\ N(\lambda) = \lambda^{n-2r} \sqrt{|\lambda^2 - h^2|} \sqrt{|\lambda^2 - k^2|} \sum_{j=0}^{r-2} a_j \lambda^{2j} & p = [2n-r+1, 2n] \end{cases} \quad (6)$$

$$\begin{aligned} K(\lambda) &= \lambda^{n-2r} \sum_{j=0}^r b_j \left(1 - \frac{\lambda^2}{h^2}\right)^j & p = [0, r] \\ L(\lambda) &= \lambda^{1-n+2r} \sqrt{|\lambda^2 - h^2|} \sum_{j=0}^r b_j \left(1 - \frac{\lambda^2}{h^2}\right)^j & p = [r+1, n] \\ M(\lambda) &= \lambda^{1-n+2r} \sqrt{|\lambda^2 - k^2|} \sum_{j=0}^r b_j \left(1 - \frac{\lambda^2}{h^2}\right)^j & p = [n+1, 2n-r] \\ N(\lambda) &= \lambda^{n-2r} \sqrt{|\lambda^2 - h^2|} \sqrt{|\lambda^2 - k^2|} \sum_{j=0}^r b_j \left(1 - \frac{\lambda^2}{h^2}\right)^j & p = [2n-r+1, 2n] \end{aligned} \quad (7)$$

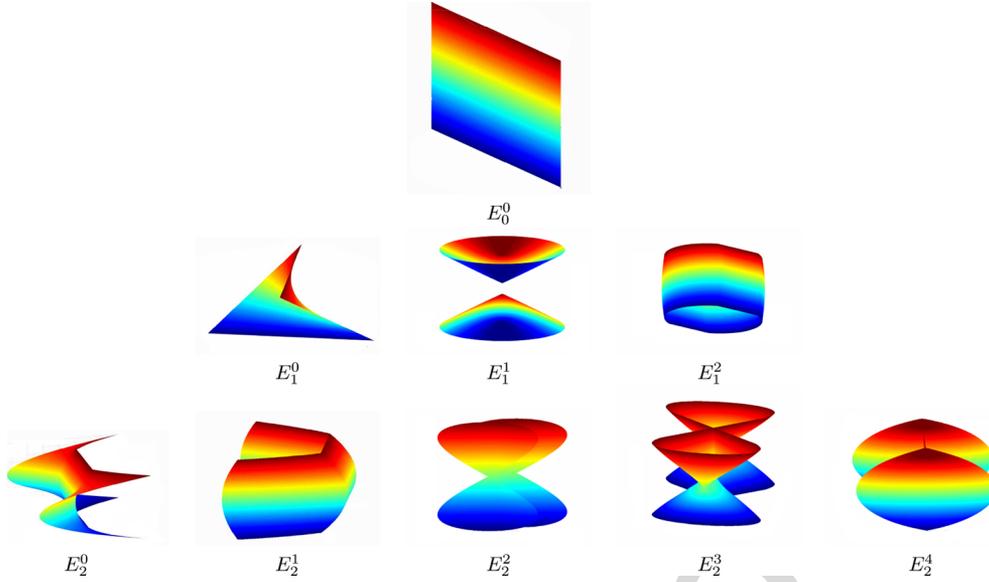


Fig. 2. Ellipsoidal harmonic expansion (in ellipsoidal coordinates).

The presence of $I_n^p(\cdot)$ ensures the stability of the result for $\lambda_1 > \alpha$, because

$$\lim_{\lambda \rightarrow \infty} E_n^p(\lambda) = \infty.$$

Fig. 2 depicts the first nine basis functions of ellipsoidal harmonics in the non-Euclidian space defined by ellipsoidal harmonics. In fact, every 3-D function is decomposed into a set of surfaces derived from the general three-variable quadratic equation (the curious reader may observe that E_1^1 is a two-sheet hyperboloid, E_2^0 is a saddle surface, and so on).

III. 3-D OBJECT DESCRIPTION USING ELLIPSOIDAL HARMONICS

In this paper, the use of the theory of ellipsoidal harmonics for 3-D object representation is proposed. The extracted ellipsoidal harmonics descriptors can then be utilized for 3-D search and retrieval applications. The major advantages of the ellipsoidal harmonics are identified as follows.

- Ellipsoids are better approximations for the majority of the 3-D objects. Spheres are the isotropic special cases of the ellipsoids and, thus, the approximation errors in some models are relatively large [33]. In contrast, an ellipsoidal implicit surface can alter its aspect ratio so as to fit better in a given model, resulting in reduced approximation errors. Thus, ellipsoidal harmonics are expected to result in a set of descriptors which has higher discriminative power when compared to similar approaches (e.g., spheres).
- Ellipsoidal harmonics result in a compact 3-D shape representation. The experiments performed proved that only few descriptors are required for accurate 3-D object representation.
- Ellipsoidal harmonics can be applied to 3-D objects that are expressed either on a surface representation or in a volumetric representation; thus, any error carried during the creation of an object can be avoided.

The key point in the extraction of the ellipsoidal harmonic descriptor is the appropriate selection of the function $O(\cdot)$ (8) which will be transformed according to the approach presented in Section II. In this virtue, three different approaches for the application of ellipsoidal harmonics are proposed: The surface ellipsoidal harmonics descriptor, which is applied in 3-D objects represented by surfaces, the volumetric ellipsoidal harmonics descriptor (VEHD), which is applied in the 3-D objects represented as volumetric functions and the generalized ellipsoidal harmonic descriptor (GEHD), applicable to any local feature of the 3-D object.

A. Ellipsoidal Harmonics Descriptor for Surface Represented 3-D Objects

The surface ellipsoidal harmonics descriptor (SEHD) can be computed as follows.

Firstly, the minimum bounding ellipsoid of the 3-D object is estimated and is considered as the reference ellipsoid of the object, i.e., the nine parameters of an ellipsoid are estimated:

$$\frac{x'^2}{a^2} + \frac{y'^2}{b^2} + \frac{z'^2}{c^2} = 1 \quad (12)$$

where

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R} & \mathbf{v} \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} \quad (13)$$

and the nine parameters are: a , b , and c which define the size of each axis of the ellipsoid, the three Euler angles represented by the 3×3 rotation matrix \mathbf{R} , and the three parameters that represent the absolute position of the ellipsoid's center in the global coordinate system (3×1 translation vector \mathbf{v}). The criterion is

the estimation of the bounding ellipsoid with minimum volume, i.e.,

$$V_{ell} = \frac{4}{3}\pi abc \rightarrow \min \quad (14)$$

and

$$\frac{x_i'^2}{a^2} + \frac{y_i'^2}{b^2} + \frac{z_i'^2}{c^2} \leq 1 \quad (15)$$

where (x_i, y_i, z_i) , $i = 1, \dots, N$ are the points of the 3-D object and V_{ell} is the ellipsoid's volume.

A very important result of this approach is the automatic estimation of the scaling, absolute, and relative position of the object in the 3-D space. Thus, a preprocessing step for normalization with respect to rotation, scaling, and translation is not required, as it is performed automatically during the minimization of (14).

Then, the surface representation is transformed so as to be expressed in the ellipsoidal coordinates. In the sequel, assuming that the input function $O(\cdot)$ in ellipsoidal harmonics analysis is $O(\lambda_1, \lambda_2, \lambda_3) = \lambda_1$, the descriptors o_n^p are computed according to (9).

It should be stated here that estimation of \mathbf{R} and \mathbf{v} can be also performed using principal component analysis of low order geometric moments. However, the resulting parameters a, b, c do not fulfill the criterion of (15). By definition, when PCA is utilized the estimated ellipsoidal parameters define the best fitted ellipsoid, which is different by the minimum volume bounding ellipsoid (Fig. 3).

B. Ellipsoidal Harmonics Descriptor for Volumetric 3-D Objects

The VEHD can be computed as follows.

Firstly, the minimum bounding ellipsoid of the 3-D object is estimated and is considered as the reference ellipsoid of the object. The estimation procedure is the same with the procedure of SEHD. Based on the reference ellipsoid, N ellipsoids are assumed at different scales and the intersection of the volume with every ellipsoid is considered. Then, every intersection is transformed so as to be expressed in the ellipsoidal coordinates and form a binary function $O(\cdot)$ in the curvilinear space of ellipsoidal coordinates. Finally, the descriptors o_n^p are computed according to (9).



Fig. 3. (a) Bounding ellipsoid (in purple) using the proposed analysis and (b) the best-fitted ellipsoid (in green) using PCA of the same object.

C. Minimum Bounding Ellipsoid of the 3-D Object

The minimum bounding ellipsoid of every object is estimated using the Computational Geometry Algorithms Library [34]. This computation estimates the absolute position of the object \mathbf{v} , the rotation matrix \mathbf{R} , and the parameters a, b, c of the object's reference ellipsoid. By doing so, normalization with respect to rotation and translation is automatically achieved, and thus, the rest of the process is invariant to rotation and translation. Moreover, using relative coordinates to the size of the ellipsoid, scaling invariance can also be achieved.

D. Computation of o_n^p Coefficients

Although the computation of o_n^p coefficient seems trivial and rather straightforward, the handling of the coordinates in a curvilinear space is rather tricky.

More specifically, a major problem during the computation of o_n^p is identified in the fact that the mapping between the Cartesian and ellipsoidal coordinates is not a one-to-one procedure. The initial solution for this problem is presented in [32], which extends the ellipsoidal coordinates $(\lambda_1, \lambda_2, \lambda_3)$ to the normalized ellipsoidal coordinates α, β, γ . However, this solution involves the computation of complete and incomplete 1st kind elliptic integrals and elliptic functions $sn(\cdot)$, $cn(\cdot)$, and $dn(\cdot)$ many times during the integration, which is time-consuming. As it is stated in [30], although the signs of λ_i are not known, the ambiguity which is being introduced can be easily overcome using the following notation:

$$E_n^p(\lambda_2, \lambda_3) = \Psi_n^p(\lambda_2, \lambda_3, x, y, z) P_n^p(\lambda_2) P_n^p(\lambda_3) \quad (16)$$

where

$$P_n^p(\lambda_i) = \sum_{j=0}^m b_j \left(1 - \frac{\lambda_i^2}{h^2}\right) \quad (17)$$

and $\Psi_n^p(\lambda_1, \lambda_2, \lambda_3, x, y, z)$ is given by (18) at the bottom of the page.

$$\Psi_n^p(\lambda_2, \lambda_3, x, y, z) = \begin{cases} (h k x)^{n-2r} & 0 \leq p \leq n-r \quad (K \text{ family}) \\ (h k x)^{1-(n-2r)} h y \sqrt{k^2 - h^2} & n-r+1 \leq p \leq n \quad (L \text{ family}) \\ (h k x)^{1-(n-2r)} k z \sqrt{k^2 - h^2} & n+1 \leq p \leq 2n-r \quad (M \text{ family}) \\ (h k x)^{n-2r} h k (k^2 - h^2) y z & 2n-r+1 \leq p \leq 2n \quad (N \text{ family}) \end{cases} \quad (18)$$

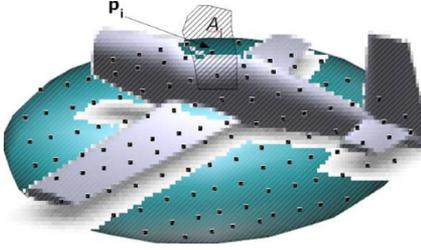


Fig. 4. Basic principle of the GEHD.

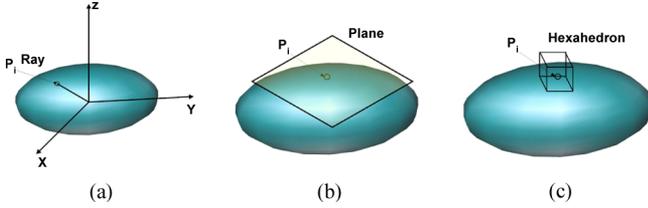


Fig. 5. Three instances of the generalized ellipsoidal harmonics using (a) a ray, (b) a tangent plane, and (c) a hexahedron.

Using (16) the computation of o_n^p coefficients overcomes the ambiguity of the non one-to-one mapping between the ellipsoidal and the Cartesian coordinates:

$$o_n^p = \int \int_S O(\lambda_1, \lambda_2, \lambda_3) \Psi_n^p(\lambda_2, \lambda_3, x, y, z) \times P_n^p(\lambda_2) P_n^p(\lambda_3) dS. \quad (19)$$

E. Generalized Ellipsoidal Harmonic Descriptor for any Kind of 3-D Object

The GEHD is the extension of SEHD and VEHD, so as the resulting descriptor captures more intuitive information.

For the computation of GEHD, firstly, N ellipsoids $E_k, k = 1, \dots, N$, confocal to the minimum bounding ellipsoid are defined and every ellipsoid E_k is sampled producing a set of sample points $P = \{\mathbf{p}_i, i = 1, \dots, S\}$. For each sample point \mathbf{p}_i , an area A_i centered in point \mathbf{p}_i is considered (Fig. 4). The part of the initial 3-D object which is included in the area A_i is analyzed using one or more local or global features (such as Fourier Descriptor, Zernike Descriptor [17], Wavelets, and Generalized Radon Transform [16]).

Let us assume that for the 3-D object Q , Q_{A_i} is the part of the 3-D object that lies in A_i . Q_{A_i} is analyzed using L different single-valued features $F_j(Q_{A_i}), j = 1, \dots, L$ (if a feature is multi-valued, then each value is a separate feature). Every function $F_j(Q_{A_i})$ can be considered as $F_j(\mathbf{p}_i)$, which can be analyzed using the ellipsoidal harmonics procedure according to (9) leading to descriptors $o_n^p(k, j)$. Fig. 4 depicts the sampling process and the selection of a single area A_i .

It should be noted that there is no limitation in the selection of the area A_i , except for $A_i = A_j \forall i \neq j$. A sphere, a box, and a N -hedron centered in \mathbf{p}_i [Fig. 5(c)], a plane tangent to \mathbf{p}_i [Fig. 5(b)], or a ray from the center of the ellipsoid that includes \mathbf{p}_i [Fig. 5(a)] are only few indicative types of the areas A_i that can be used. Furthermore, there is no limitation in the selection of the features computed in every Q_{A_i} .

A curious reader may notice that both VEHD and SEHD are special cases of the generalized ellipsoidal harmonic descriptor, where the functionals are the volume density and the distance from the center of the ellipsoid for VEHD and SEHD, respectively.

For the needs of this paper, the area A_i has been selected to be a cubic box, sized $5 \times 5 \times 5$ voxels, centered at \mathbf{p}_i . The selected functionals are appropriately selected coefficients of the 3-D wavelet transform, using Daubechies filters.

F. Example of Ellipsoidal Harmonics Descriptor Extraction

Let us assume that M is the 3-D object presented in Fig. 6. In this section, the process of extracting the VEHD will be thoroughly presented. The other two instances of ellipsoidal harmonic descriptors (SEHD and VEHD) are extracted in a similar manner.

The VEHD extraction process of M (Fig. 6, top-left) is as follows.

- The minimum bounding ellipsoid of the 3-D object is estimated according to [34] and is considered as the reference ellipsoid (Fig. 6, top-right).
- Then, 24 ellipsoids of different scales are defined and their intersections with the 3-D object are extracted. Fig. 6 (second line) depicts two intersections as examples.
- The i th intersection ($i = 1, \dots, N$) is a function defined on the surface of an ellipsoid. (Fig. 6, second line, presents two indicative intersections). It is transformed to the non-Euclidean space of ellipsoidal coordinates by solving (2) for every 3-D point forming a function $O_i(\cdot)$.
- Functions $O_i(\cdot)$ ($i = 1, \dots, N$) are analyzed in their ellipsoidal harmonics coefficients (Fig. 6, last line), resulting in the descriptor vector.

In the last line of the example presented in Fig. 6 is depicted the differences in the ellipsoidal harmonic expansion of two different functions. In cases where the expanded function does not present variations, the significant coefficients are few. The significant coefficients are more, when the function presents significant variations.

IV. COMBINED SPHERICAL—ELLIPSOIDAL HARMONIC DESCRIPTION

An innovative combination of spherical and ellipsoidal harmonics can be produced by performing multiple computation of ellipsoidal harmonics by placing the bounding ellipsoid in various orientations θ, ϕ of spherical coordinates and computing the spherical harmonic coefficients of every ellipsoidal coefficient (Fig. 7). The reference ellipsoid is rotated and ellipsoidal harmonics are computed at various orientations. Then, for every o_n^p coefficient, the spherical harmonics transformation is performed. The basic motivation behind this combination is to produce a novel 3-D feature that inherits both the “directional” information captured by ellipsoidal harmonics and the rotation invariant properties of spherical harmonics.

Specifically, the descriptor extraction procedure is the following: Initially, the nine parameters of the minimum bounding ellipsoid are estimated, according to the methodology presented in the previous section. The ellipsoid is placed in the appropriate position; however, rotation normalization is not

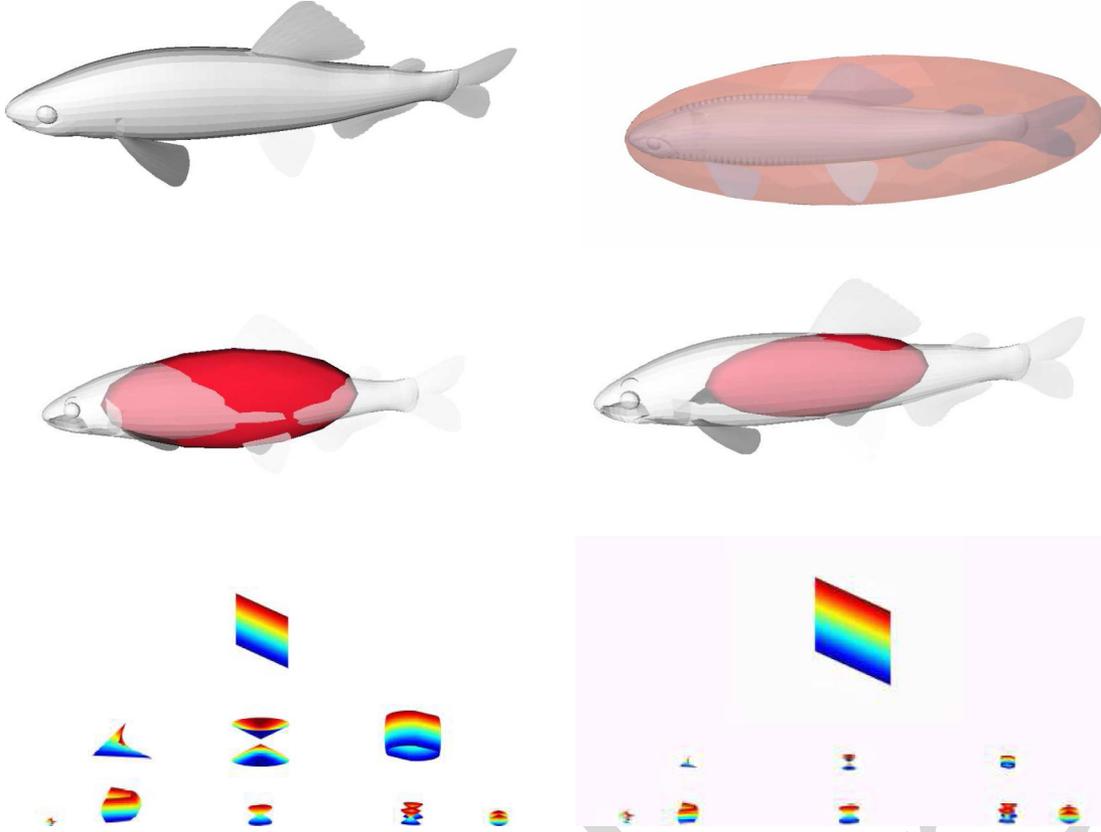


Fig. 6. Extraction process: The minimum bounding ellipsoid of the initial object (top left) is estimated (top right). A function defined in an ellipsoid is defined, in this example two intersections of the ellipsoid with 3-D object are presented (second line). The dark red areas of the ellipsoid are valued with zero and the light areas are valued with a non-zero value. These function is analyzed according to ellipsoidal harmonics basis functions as can be seen in the last line.

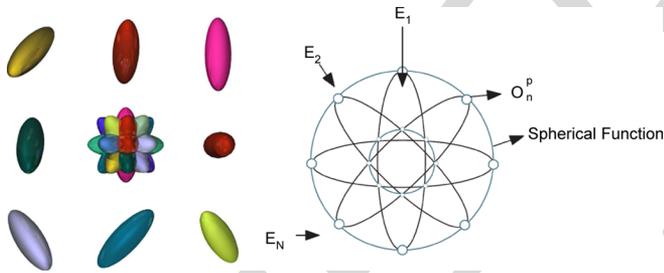


Fig. 7. Combining spherical and ellipsoidal harmonics.

performed. In the sequel, various orientations of the reference ellipsoid are considered. For each orientation, ellipsoidal harmonic descriptors are computed, using either VEHD, GEHD, or SEHD (Fig. 7). Every ellipsoidal harmonic descriptor from all orientations forms a spherical function defined in θ, ϕ of spherical coordinates. Finally, every spherical function is analyzed according to spherical harmonics analysis, and the resulting coefficients are forming the descriptor vector of the 3-D object, the combined spherical and ellipsoidal harmonics descriptor (SH + EHD).

It should be noted that every ellipsoidal harmonic descriptor can be extended to the combined spherical-ellipsoidal harmonics descriptor. The major advantage of the latter approach

is the computation of native rotation invariant descriptors, which overcome the errors posed from incorrect rotation estimation of the initial 3-D object.

Lemma: Combined Spherical-Ellipsoidal Harmonic Description Is Rotation Invariant:

Proof: Let O be a 3-D object and $o_p^n(\theta, \phi)$ its ellipsoidal harmonic descriptors, when the reference ellipsoid major axis is placed on θ, ϕ of spherical coordinates.

Let also O^R be the same 3-D object rotated around an arbitrary axis by R and ${}^R o_p^n(\theta, \phi)$ its ellipsoidal harmonic descriptors. It is obvious that $\forall(\theta, \phi) \exists(\theta_R, \phi_R)$ which is the result of the rotating the axis (θ, ϕ) around the same axis by R , where $o_p^n(\theta, \phi) = {}^R o_p^n(\theta_R, \phi_R)$ and that $o_p^n(\theta + \delta\theta, \phi + \delta\phi) = {}^R o_p^n(\theta_R + \delta\theta, \phi_R + \delta\phi)$. In fact, the function ${}^R o_p^n(\cdot)$ is a rotated version of $o_p^n(\cdot)$.

Let $T[\cdot]$ be the transformation of a spherical function to its rotation invariant spherical harmonic expansion. Then, $T[{}^R o_p^n(\cdot)] = T[o_p^n(\cdot)]$.

V. MATCHING METHOD

Let us assume that two 3-D objects O_1 and O_2 are described using the ellipsoid harmonics descriptors $o_n^{p(1)}$ and $o_n^{p(2)}$, respectively. In order to calculate a similarity metric between the

two objects, the normalized Minkowski L_1 distance has been utilized:

$$L_1(O_1, O_2) = \sum_{n=0}^{n=N_h-1} \sum_{p=0}^{p=2n} \left| o_n^{p(1)} - o_n^{p(2)} \right| \quad (20)$$

where

$$o_n^{p(i)} = \frac{o_n^{p(i)}}{\sum_{n=0}^{n=N_h-1} \sum_{p=0}^{p=2n} o_n^{p(i)}} \quad (21)$$

and N_h is the maximum order of harmonics.

VI. COMPUTATIONAL ASPECTS

The computational cost of the SEHD and VEHD methods can be analyzed as follows.

1) *Minimum Bounding Ellipsoid Estimation*: This part of the procedure is crucial, because it defines the basis of the curvilinear coordinate system. This computation is performed only once and according to the CGAL [34], its complexity is $O(9k(\epsilon^{-1} + \ln 3 + \ln \ln k))$, where k is the number of model points. In practice, this part of the procedure is completed in less than 0.01 s for a typical 3-D object in the testing machine.

2) *Computation of o_n^p Coefficients*: The computation of the coefficients depends on two factors.

- The integration algorithm: For the integration, the traditional brute-force approach has been followed (i.e., summation of the values in every sample). Thus, a smart sampling approach has been adopted, where $S = 5200$ samples are uniformly selected over the surface of the ellipsoid for VEHD and SEHD. Using more sample points, the computational time is increased, while there is no serious affect in the retrieval performance of the approach. For the GEHD, the sampling rate is $S = 1300$ samples per ellipsoid, which is smaller due to the fact that GEHD is based on an area around the sample point, resulting in local descriptors that comprise information of the surrounding area of the sample.
- The algorithm for polynomial $P_n^p(\cdot)$ computation: one of the most efficient algorithms has been adopted for the needs of this paper, the simple Horner's rule that requires exactly n multiplications and n additions for every value of $P_n^p(\cdot)$.

The computational complexity of the proposed approach SH + EHD is due to:

- computation of lame polynomials (once);
- computation of ellipsoidal harmonics coefficients for all possible directions of the ellipsoid;
- computation of spherical harmonics (once).

A major advantage of the SH + EHD is identified by the fact that the computation of ellipsoidal harmonics coefficients for all possible directions is a procedure which can be easily performed using multithreaded processing.

TABLE III
COMPUTATION TIMES FOR THE VARIOUS INSTANCES
OF ELLIPSOIDAL HARMONIC DESCRIPTOR

Method	Mean Computation Time
SEHD	1.02 secs
VEHD	1.57 secs
GEHD	1.95 secs
SH+EHD	6.37 secs
REXT	0.58 secs
GEDT	2.4 secs
LFD	2.77 secs

VII. EXPERIMENTAL RESULTS

The proposed approach has been evaluated for its retrieval performance using the Princeton Shape Benchmark (PSB) [35], the Engineering Shape Benchmark (ESB) [36] and the ITI 3-D model's database [16] (the ITI database can be downloaded from <http://www.victory-eu.org>) and has been compared to well-known approaches of Gaussian Euclidean distance transform (GEDT), which is based on the comparison of a 3-D function, whose value at each point is given by composition of a Gaussian with the Euclidean distance transform of the surface [9], the Light field descriptor (LFD) where a representation of a model as a collection of images rendered from uniformly sampled positions on a view sphere is utilized [11], and the Radialized spherical extent function (REXT) where a collection of spherical functions giving the maximal distance from center of mass as a function of spherical angle and radius is utilized [12]. The retrieval accuracy has been evaluated using the precision-recall diagrams, where precision is defined as the ratio of the relevant retrieved objects and the total number of the retrieved objects, and recall is the ration of the relevant retrieved objects and the total relevant objects in the database. For the ESB and PSB datasets, which adopt multilevel classification schemes, an object is considered similar to the query if both objects belong to the same subclass.

The experiments performed using an Intel Core Duo running at 1.5 GHz per core with 2 GB RAM, running Windows XP. The source code has been compiled using Microsoft Visual Studio 8.0 SP1, utilizing multithreaded processing techniques. The results concerning the approaches of [9], [11], and [12] have been derived using the executables provided by the authors of corresponding articles.

The volumetric ellipsoidal harmonic descriptor and the generalized ellipsoidal harmonic descriptor have been computed for 3-D objects that are expressed in a $64 \times 64 \times 64$ cubic voxel grid. The number of the ellipsoids has been selected to be $N = 24$ (for VEHD) and $N = 12$ (for GEHD) and the maximum degree of expansion $N_h = 6$. The combined spherical-ellipsoidal descriptor (SH + EHD) has been computed for $N = 5$ ellipsoids and 342 different orientations.

Firstly, the computational complexity of the proposed approach has been evaluated in terms of execution time. Table III presents the average computational time required for the computation for the various instances of the ellipsoidal harmonics descriptors and the methods of [9], [11], and [12].

The proposed methods require slightly more execution time than the methods presented in the literature. However, the latter cannot be considered as a major drawback because the execution

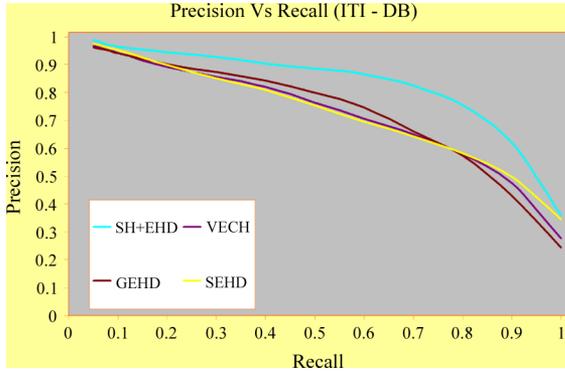


Fig. 8. Comparative precision-recall diagrams for the various instances of EH in the ITI database.

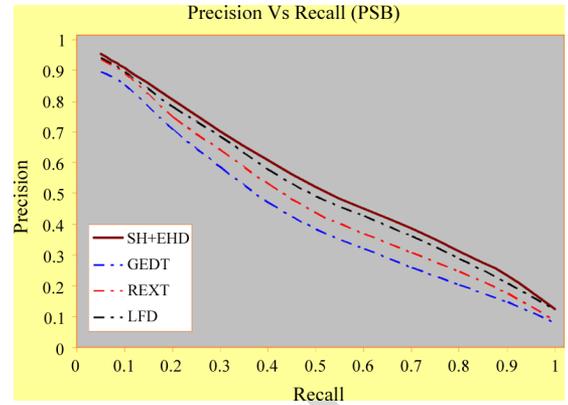


Fig. 10. Comparative precision-recall diagrams for the Princeton Shape Benchmark.

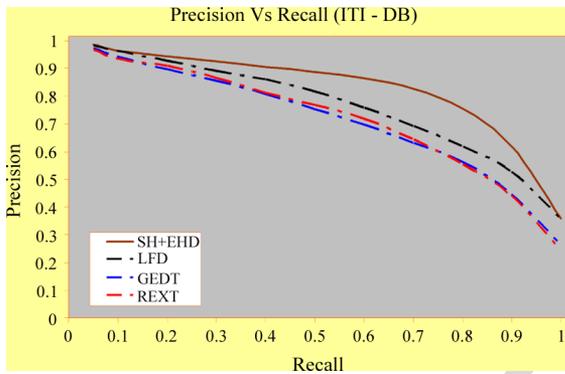


Fig. 9. Comparative precision-recall diagrams for the ITI database.

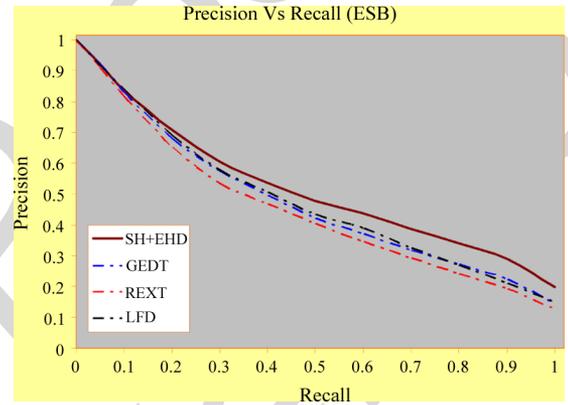


Fig. 11. Comparative precision-recall diagrams for the Engineering Shape Benchmark.

times are small enough to be used in practical search problems, and the hardware is continuously improved leading in smaller execution times. Moreover, the descriptor extraction takes place only once for each 3-D object and the descriptors are stored in the database. During retrieval, the descriptors are retrieved from the database and are compared to the query's descriptors.

In the sequel, the rotation invariance of SH + EHD descriptor is experimentally verified. A set of sample objects has been arbitrarily rotated and the SH + EHD descriptors for both original and rotated sets have been computed. The maximum difference between the original and rotated version of the same object was 0.5%. The small differences are expected in practice due to the discrimination of the continuous spherical harmonic process.

Fig. 8 depicts the comparative performance of the instances of the ellipsoidal harmonic descriptor on the ITI database. It is observed that GEHD's retrieval accuracy outperforms VEHD's and SEHD which are comparable. This can be explained by the fact that both VEHD and SEHD are trivial instances of GEHD. When another kind of information is considered (e.g., the 3-D wavelet transform utilized in this paper), GEHD provides slightly better results. It is also obvious that the combined spherical-ellipsoidal harmonic descriptor outperforms the simple ellipsoidal and the generalized ellipsoidal harmonic descriptors and, thus, is selected to be compared to the other approaches. Figs. 9–11 depict the comparative performance of the best instance (SH+EHD) of the ellipsoidal harmonic descriptor

for the three different databases compared to the approaches of [9], [11], and [12]

By comparing the precision-recall diagrams for the three different databases, it is obvious that all the approaches have completely different behavior in different databases. The latter basically happened due to the different nature of each database, the number of the 3-D objects it contains, and the way that the 3-D objects have been classified.

The efficiency of ellipsoidal harmonics is depicted in the combined spherical-ellipsoidal harmonic descriptor, which outperforms all competitive approaches and any other ellipsoidal harmonic descriptor. The combined spherical-ellipsoidal harmonic descriptor combines the rotation invariant features provided by the spherical harmonic analysis, with directional features captured by the ellipsoidal harmonic analysis, and thus, the resulting descriptor has significantly better retrieval performance when compared to other approaches.

It is very interesting that the performance of the proposed approach is significantly greater than the other approaches for Recall > 0.4 in the CAD database (Fig. 11) and in the ITI database (Fig. 9). The latter means that all relevant objects will be presented earlier to the user when compared to other approaches. Both ITI and ESB are mainly composed of objects

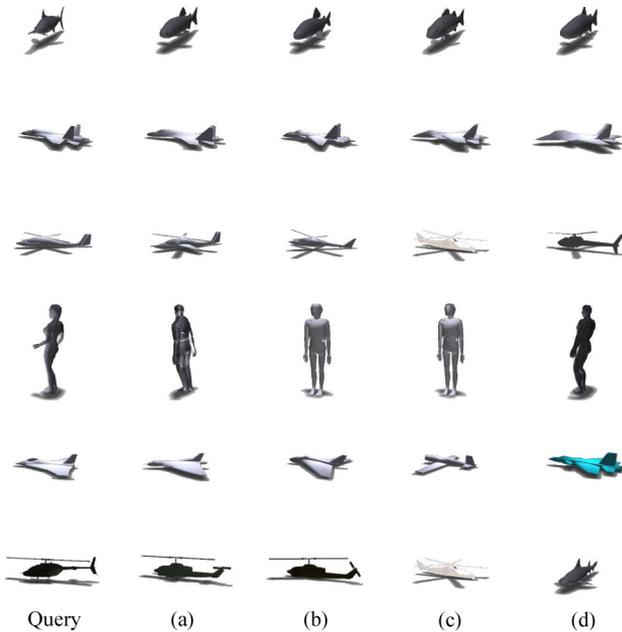


Fig. 12. Indicative retrieved results. The first column depicts the query 3-D object while the rest are the first four retrieved objects.

having mainly directional information, and thus, the combination of the rotation invariant features provided by the spherical harmonic analysis, with directional features captured by the ellipsoidal harmonic analysis, achieve better retrieval efficiency in these databases.

In Fig. 12, selected retrieved results are shown. The first column is the query model, while the rest are the first four retrieved results. From the presented results, the geometric similarity between the query and the retrieved objects is obvious; however, there are semantic differences in some retrieved results, e.g., in the last two lines of Fig. 12. When the query is a “delta plane”, a conventional plane is ranked third, and when the query is a helicopter, a fish is ranked in the fourth position. Results of this kind are expected, since neither topology nor high-level information has been taken into account that could semantically discriminate these 3-D objects. From a geometric perspective, the major difference between a helicopter and a fish is located in the helicopter’s wings (the different scaling has been discarded during normalization). In the proposed approach, intersections of the 3-D object with many ellipsoids are utilized to extract the shape descriptors. When comparing the intersections of a helicopter to the intersections of a fish, only few intersections differ significantly, while a great majority is highly similar, and thus, the mean dissimilarity score will be low. Additionally, two objects of the same class may be normalized in slightly different scale, and thus, few intersections will present significantly high dissimilarity. As a result, scores between some helicopters can potentially be comparable to the scores between a helicopter and a fish, resulting in retrievals similar to the one presented in the last line of Fig. 13, where a fish is ranked fourth when the query is a helicopter.

VIII. CONCLUSIONS

In this paper, various novel descriptors based on ellipsoidal harmonics were introduced. The combination of spherical and ellipsoidal harmonics results in a more discriminative descriptor set which is capable of performing robust 3-D content-based search and retrieval for online applications. The experimental results proved the efficiency of the proposed descriptors in performing geometry-based 3-D object search and retrieval. Although geometry-based content retrieval provides very good results, the geometry of a 3-D object may not always provide the semantically similar results. In these cases, the geometry-based results should be combined in a semantic-based framework where the system is enhanced with external knowledge in order to improve the retrieval performance.

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Athanasios Mademlis was born in Thessaloniki, Greece, in 1980. He received the Diploma degree in electrical and computer engineering and the Ph.D. degree in electrical and computer engineering from Aristotle University of Thessaloniki, Thessaloniki, Greece, in 2004 and 2009, respectively.

Dr. Mademlis is a member of the Technical Chamber of Greece.

Petros Daras (M'XX) **AUTHOR: WHAT YEAR?** was born in Athens, Greece, in 1974. He received the Diploma degree in electrical and computer engineering, the M.Sc. degree in medical informatics, and the Ph.D. degree in electrical and computer engineering from the Aristotle University of Thessaloniki, Thessaloniki, Greece, in 1999, 2002, and 2005, respectively.

He is a Senior Researcher at the Informatics and Telematics Institute. His main research interests include computer vision, search and retrieval of 3-D objects, and multimedia search engines. His involvement with those research areas has led to the coauthoring of more than 12 papers in refereed journals and more than 40 papers in international conferences. He has served as a regular reviewer for a number of international journals and conferences. He has been involved in more than 15 European and National research projects.

Dr. Daras is a member of the Technical Chamber of Greece.

Dimitrios Tzovaras received the Diploma degree in electrical engineering and the Ph.D. degree in 2-D and 3-D image compression from Aristotle University of Thessaloniki, Thessaloniki, Greece, in 1992 and 1997, respectively.

He is a Senior Researcher in the Informatics and Telematics Institute of Thessaloniki. Prior to his current position, he was a Senior Researcher on 3-D imaging at the Aristotle University of Thessaloniki. His main research interests include virtual reality, assistive technologies, 3-D data processing, medical image communication, 3-D motion estimation, and stereo and multiview image sequence coding. His involvement with those research areas has led to the coauthoring of more than 35 papers in refereed journals and more than 80 papers in international conferences. He has served as a regular reviewer for a number of international journals and conferences. Since 1992, he has been involved in more than 40 projects in Greece, funded by the EC, and the Greek Secretariat of Research and Technology.

Dr. Tzovaras is a member of the Technical Chamber of Greece.

Michael Gerassimos Strintzis (M'70–SM'80–F'04) received the Diploma degree in electrical engineering from the National Technical University of Athens, Athens, Greece, in 1967, and the M.A. and Ph.D. degrees in electrical engineering from Princeton University, Princeton, NJ, in 1969 and 1970, respectively.

He then joined the Electrical Engineering Department at the University of Pittsburgh, Pittsburgh, PA, where he served as Assistant Professor (1970–1976) and Associate Professor (1976–1980). Since 1980, he has been a Professor of electrical and computer engineering at the University of Thessaloniki, Thessaloniki, Greece, and, since 1999, Director of the Informatics and Telematics Research Institute, Thessaloniki. His current research interests include 2-D and 3-D image coding, image processing, biomedical signal and image processing, and DVD and Internet data authentication and copy protection.

Dr. Strintzis has served as an Associate Editor for the IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS FOR VIDEO TECHNOLOGY since 1999. In 1984, he was awarded one of the Centennial Medals of the IEEE.

Ellipsoidal Harmonics for 3-D Shape Description and Retrieval

Athanasios Mademlis, Petros Daras, *Member, IEEE*, Dimitrios Tzovaras, and Michael Gerassimos Strintzis, *Fellow, IEEE*

Abstract—In this paper, a novel approach for 3-D Shape description and retrieval based on the theory of ellipsoidal harmonics is presented. Four novel descriptors are introduced: the surface ellipsoidal harmonics descriptor, which concerns 3-D objects that are described as polygonal surfaces; the volumetric ellipsoidal harmonics descriptor, which is applicable to volumetric 3-D objects; the generalized ellipsoidal harmonics descriptor that is applied to any local 3-D object descriptors; and, finally, the combined ellipsoidal-spherical harmonics descriptor, which leads to a compact and powerful descriptor that inherits the advantages of both approaches: the rotation invariance properties of the spherical harmonics and the directional information enclosed in ellipsoidal harmonics. Experimental results performed using well-known 3-D object databases prove the retrieval efficiency of the proposed approach.

Index Terms—3-D object retrieval, ellipsoidal harmonics, shape description.

I. INTRODUCTION

THE 3-D object retrieval is a relatively new and very challenging research field and a major effort of the research community has been devoted to the formulation of accurate and efficient 3-D object search and retrieval algorithms. In the new 3-D era, the growth of the Internet along with the recent progress in computer's graphical units and the development of friendly and easy-to-use 3-D content creation tools have led to the creation of huge repositories with 3-D objects. In this new environment, the demand for efficient tools that can quickly and accurately retrieve the desired 3-D content is emerging.

Manuscript received July 28, 2008; revised August 04, 2009. This work was supported in part by the VICTORY project funded by the European Commission and in part by the PENED project co-funded 75% from European Commission and 25% by the Greek Secretariat for Research and Technology. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Mohan S. Kankanhalli.

A. Mademlis is with the Information Processing Laboratory, Department of Electrical and Computer Engineering, Aristotle University of Thessaloniki, Thessaloniki, Greece (e-mail: mademlis@iti.gr).

P. Daras and D. Tzovaras are with the Informatics and Telematics Institute, Centre for Research and Technology Hellas, Thessaloniki, Greece (e-mail: daras@iti.gr; Dimitrios.tzovaras@iti.gr).

M. G. Strintzis is with the Information Processing Laboratory, Department of Electrical and Computer Engineering, Aristotle University of Thessaloniki, Thessaloniki, Greece, and also with the Informatics and Telematics Institute, Centre for Research and Technology Hellas, Thessaloniki, Greece (e-mail: strintzis@eng.auth.gr).

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Digital Object Identifier 10.1109/TMM.2009.2032690

A. Related Work

In the last few years, a lot of work has been performed in the area of accurate 3-D shape description for search and retrieval applications. The presented approaches can be classified into four major categories: global feature-based approaches, local feature-based approaches, topology-based approaches, and view-based approaches.

The global feature-based approaches, which are the first methods that appear in the field of 3-D search and retrieval, aim to capture the geometry of the whole object in a single representation (usually a vector), using various methods: primitive shape features [1], [2], moments (Krawtchouk [3], Zernike [4]), transformations, etc. Local feature-based approaches attempt to describe the complete 3-D object using local features. The majority of the local feature-based approaches describes the complete geometry using histograms; however, few attempts to further exploit the local information have been recently presented.

In topology-based approaches, the main feature of the 3-D object is formed by its topology instead of its geometry. The topology is usually represented in the form of a graph, and the matching is performed using dedicated graph matching techniques. The view-based approaches cannot be classified as native 3-D description methods. The object is decomposed to a collection of 2-D views and the most powerful image descriptors are utilized. Table I summarizes some important 3-D object retrieval approaches. For more sophisticated analysis, the reader is referred to the 3-D search and retrieval reviews [5], [6]. More recently, the field of 3-D shape search and retrieval has attracted more researchers and the competition on the field has been increased significantly, mainly due to the Shape Retrieval Contest (SHREC), organized each year by the consortium of Aim@Shape EU-funded project [7].

All of the existing approaches present advantages and drawbacks. Topology-based approaches are the only approaches that can capture topological information; however, few methods exist that equally rely on both topology and geometry. Additionally, the majority of the topology-based approaches cannot easily generalize to all kinds of 3-D objects and they are very sensitive to minor shape changes (the topology can be seriously altered); thus, their applicability is limited to few classes of 3-D objects. Local feature-based approaches focus on acquiring highly discriminant local representations, which are usually integrated in a single (or multiple) histogram(s) and thus, their discriminative power is seriously affected in the majority of the approaches. View-based approaches provide very reliable 3-D shape representation; however, due to their nature, they

TABLE I
RELATED WORK PRESENTED IN THE LITERATURE

Global Feature	Primitive shape features: Volume, area, moments [1], bounding box, cords, moments, wavelets [14], convex hull features (crumpliness, packing and compactness) [2], Spatial Maps of volumetric features [15], [10] Transformations: Generalized Radon Transform [16], Spherical Harmonics [9], 3D Zernike [17], complex spherical functions [18], [19], 3D angular radial transform (ART) [20]
Local Feature	Forming Histograms: MPEG-7 shape index [21], Extended Gaussian Images [22], Complex Extended Gaussian Images [23], Space partitioning histograms [10] Spin Image Singatures [24], Priority-Driven Search [25].
Topology Based	Curve Skeletons [26], [27], Reeb Graphs[28], Medial Surfaces [29]
View Based	Light-Field Descriptor [11]

TABLE II
COMPARISON OF RELATED WORK PRESENTED IN THE LITERATURE

Method	Invariance				Generality
	Rotation	Translation	Scale	Articulation	
[16]	PCA	Normalization	Yes	No	Yes
[9]	Yes	Normalization	Normalization	No	Yes
[17]	Yes	Normalization	Normalization	No	Yes
[20]	PCA	Normalization	Normalization	No	Yes
[25]	Yes	Yes	Yes	Yes	Yes
[28]	Yes	Yes	Yes	Yes	Yes
[29]	Yes	Yes	Yes	Yes	No
[26]	Yes	Yes	Yes	Yes	No
[11]	Search	Normalization	Normalization	No	Yes

Legend:

PCA: Rotation Invariance is achieved using Principal Component Analysis. Normalization: A normalization step is required before applying the method. For scale and translation normalization is robust. Search: the invariance is achieved during search using complex techniques.

are unable to capture features that cannot be seen from the selected points of view. Also, their time performance during retrieval can be considered as a serious disadvantage (N to N matching is required). Global feature-based approaches are more reliable when compared to topology-based and local feature-based approaches due to their ability to capture mainly the global geometry of the 3-D object discarding local features, i.e., minor shape changes between two objects do not seriously affect the global shape description. The latter is suitable for the general purpose 3-D object retrieval applications where the retrieval is based on global shape similarity. However, it can also be considered as a potential drawback in some cases, when there are objects of the same class that can dramatically change their shape (non-rigid objects or, usually, articulated objects), or objects from different classes that present major similarities (e.g., spindle of the helicopters has similar geometry to a fish).

There are also other problems that pose obstacles in the efficiency of the existing global-based approaches, such as the 3-D object's degeneracies (e.g., holes, missing polygons, hidden polygons), the 3-D object's pose normalization, the retrieval accuracy, etc. The first problem is usually tackled successfully by applying a triangulation algorithm (e.g., Delaunay triangulation) or a hole filling algorithm [8]. Concerning the pose normalization problem, there are two widely acceptable solutions presented in the literature: the natively rotation invariant description of the 3-D object (e.g., using spherical harmonics [9] histogram-based descriptors [10]) or natively rotation invariant matching (e.g., light field descriptor [11]) and the rotation normalization of the object in a preprocessing step. Both approaches present major advantages and serious drawbacks: Firstly, the vast majority of the utilized rotation normalization approaches are based on the PCA (e.g., continuous PCA [12]).

Although algorithms that utilize pose normalization using PCA usually result in descriptors with higher discriminative power, some similar objects are not usually normalized in a similar manner [13]. In contrast, natively rotation invariant object description [9] usually involves an integration-like technique which leads to descriptors which are not adequately discriminant [12]. Table II summarizes the properties of some approaches presented in the literature.

B. Motivation and Contributions of the Proposed Work

As it was earlier mentioned, one of the major problems of the global approaches is the trade-off between rotation invariance and highly discriminative shape information. In this paper, new geometric descriptors are proposed, which are based on the ellipsoidal harmonics. Ellipsoidal harmonics offer a compact and discriminative object representation that is appropriate for 3-D content-based search and retrieval. The proposed approach can be utilized using both surface-based and volumetric-based 3-D object representation and is invariant under scaling and translation of the 3-D object, using relative distances to the parameters of the bounding ellipsoid. For rotation normalization, an appropriate normalization approach is introduced (without using the well-known traditional PCA). Then, ellipsoidal harmonics analysis is extended and applied to local 3-D descriptors, leading to the generalized ellipsoidal harmonics descriptor. Finally, the directional information of the ellipsoidal harmonic descriptors is combined with spherical harmonics in order to produce a natively rotation invariant descriptor that inherits the properties of both descriptors.

The major contributions of the proposed approach are the following.

- Compact representation: The resulting descriptor set is very compact, i.e., the descriptor vector dimensionality is small.
- Better object approximation: The approximation of a 3-D object using ellipsoids is better than using spheres.
- The method is applicable on both volumetric and surface-expressed 3-D objects.
- The method can easily be generalized in order to utilize any local descriptor.
- The method can easily be combined with spherical harmonics in order to produce natively rotation invariant descriptors.
- The proposed methods are not sensitive to minor shape changes.

The rest of this paper is organized as follows: In Section II, the theory of ellipsoidal harmonics is briefly reviewed. In Section III, the various instances of ellipsoidal harmonics descriptors are described and in Section IV, ellipsoidal harmonics are combined with spherical harmonics. The utilized matching method is presented in Section V, and the experimental results are given in Section VII. Finally, the conclusions are drawn in Section VIII.

II. ELLIPSOIDAL HARMONICS

The ellipsoidal harmonics are special functions that have been utilized in the field of astronomy [30] for describing surrounding force-fields of non-spherical objects. In this paper, the theory of ellipsoidal harmonics is adapted and applied to the field of 3-D shape analysis and description. The basic motivation behind the selection of ellipsoidal harmonics for 3-D object description relies on the intuition that an ellipsoid forms a better approximation of the shape of a 3-D object when compared to the approximation using spheres. The theoretical problem of the ellipsoidal harmonics has been targeted by mathematicians for many years in the early 1900s. Thus, many variations in the notation of ellipsoidal harmonics can be found in the literature (e.g., [31], [32]). For the needs of this paper, the notation utilized in [31] has been adopted, due to its simplicity. For the sake of completeness, the theory of ellipsoidal harmonics is briefly reviewed.

A. Ellipsoidal Coordinates

Intuitively, one of the basis functions in the space of ellipsoidal coordinates has to be an ellipsoid. The latter is verified by the definition of the ellipsoidal coordinates, which are defined using a reference ellipsoid with axes length a, b, c :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1. \quad (1)$$

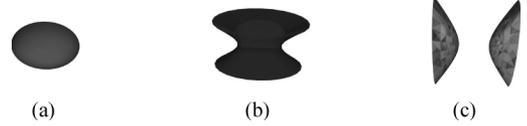


Fig. 1. Ellipsoid $\lambda_1^2 = const$ (a), the one-sheet hyperboloid (b) $\lambda_2^2 = const$, and the two-sheet hyperboloid (c) $\lambda_3^2 = const$.

Without any loss of the generality of the approach, it is assumed that $a \geq b \geq c$. By setting $\lambda_i^2 = a^2$, $h^2 = a^2 - b^2$, and $k^2 = a^2 - c^2$, the above equation can be transformed to

$$\frac{x^2}{\lambda_i^2} + \frac{y^2}{\lambda_i^2 - h^2} + \frac{z^2}{\lambda_i^2 - k^2} = 1. \quad (2)$$

For any given point in the Euclidean 3-D space $P(x, y, z)$, (2) with respect to λ_i^2 , has three discrete solutions, $\lambda_1^2 \in [k^2, +\infty)$, $\lambda_2^2 \in [h^2, k^2]$, and $\lambda_3^2 \in [0, h^2]$, which are called ellipsoidal coordinates. The ellipsoidal coordinates form an orthogonal basis for a curved space that is created by homofocal ellipsoids. For a given (h, k) , the family of ellipsoids, obtained for different values of λ_i^2 , are homofocal. The equations $\lambda_1^2 = const$, $\lambda_2^2 = const$, and $\lambda_3^2 = const$ define an ellipsoid, a hyperboloid of one sheet, and a hyperboloid of two sheets, respectively (Fig. 1). Although the mapping between ellipsoidal and Cartesian coordinates is not one-to-one (because of the squares), this problem can easily be avoided (further information is provided in Section III-D).

B. Lamé Polynomials

Ellipsoidal harmonics are the solutions of the Laplace equation in ellipsoidal coordinates $\nabla^2 V = 0$. The main advantages of this approach (from the mathematical scope of the problem) is spotted in the fact that the solutions of Laplace's equation are orthogonal and separable, i.e., see equation (3) and (4) at the bottom of the page, where $\delta(\cdot)$ is the Kronecker δ function. The Laplace equation in the space of ellipsoidal coordinates is simplified to [30]

$$\begin{aligned} & (\lambda_i^2 - h^2)(\lambda_i^2 - k^2) \frac{d^2 E_n(\lambda_i)}{d\lambda_i^2} \\ & + \lambda_i(2\lambda_i^2 - h^2 - k^2) \frac{dE_n(\lambda_i)}{d\lambda_i} \\ & + (p - n(n+1)\lambda_i^2) E_n(\lambda_i) = 0. \end{aligned} \quad (5)$$

For every n , (5) has exactly $2n$ solutions, the polynomials $E_n^p(\cdot)$ (are known both as ellipsoidal harmonics and lamé polynomials), where $n = 0 \dots \infty$ and $p = 0 \dots 2n$. $E_n^p(\cdot)$ form a complete set of basis functions of the curved space of ellipsoidal

$$\mathbf{E}_n^p(\lambda_1, \lambda_2, \lambda_3) = E_n^p(\lambda_1) E_n^p(\lambda_2) E_n^p(\lambda_3) \quad (3)$$

$$\int \int E_n^p(\lambda_2) E_m^q(\lambda_2) E_n^p(\lambda_3) E_m^q(\lambda_3) dS = \delta(m - n, p - q) \quad (4)$$

coordinates. The n th-degree lame polynomials are n th-degree polynomials of λ^2 and can be classified in four families according to (6), where $r = \lfloor n/2 \rfloor$. There are $r + 1$ polynomials that belong to the family K , $n - r$ polynomials that belong to the families L and M , and r polynomials that belong to family N . See (6) at the bottom of the page.

Due to the polynomial nature of lame polynomials (6) and the values of λ_1, λ_2 and λ_3 , the values of a_j are rapidly vanishing as j increases. Although, theoretically, this is not a major issue, in practice, the limited accuracy of existing computational systems results in inaccurate values of a_j for $j > A$, where A depends on the arithmetic precision utilized. In order to prevent the latter, the above (6) are usually transformed in the form (7) [30]. See (7) at the bottom of the page.

In (7), the equations involve polynomials of $f = (1 - \lambda^2/h^2)$, which is valued $0 \leq f \leq 1$ for $\lambda = \lambda_3$ and $1 - k^2/h^2 \leq f \leq 0$ for $\lambda = \lambda_2$. Using this computation scheme, the values of parameters b_j are not quickly vanishing, allowing more accurate results for higher order polynomials.

Both parameters a_j and b_j for every ellipsoidal harmonic degree are easily computed using appropriate eigenanalysis. Detailed computational issues of ellipsoidal harmonics are above the scope of this paper and the reader is referred to [30] and [31].

C. Ellipsoidal Harmonics Expansion

Taking advantage of the lame's polynomials orthogonality, every 3-D function defined in the space of ellipsoidal coordi-

nates $O(\lambda_1, \lambda_2, \lambda_3)$ can be expressed as a weighted sum of the lame polynomials, i.e.,

$$O(\lambda_1, \lambda_2, \lambda_3) = \sum_{n=0}^{\infty} \sum_{p=0}^{2n} o_n^p E_n^p(\lambda_1) E_n^p(\lambda_2) E_n^p(\lambda_3) \quad (8)$$

where

$$o_n^p = \int \int_S O(\lambda_1, \lambda_2, \lambda_3) E_n^p(\lambda_2) E_n^p(\lambda_3) dS. \quad (9)$$

The o_n^p values can fully characterize every function defined in the space of ellipsoidal coordinates for $\lambda_1 \leq \alpha$. For $\lambda_1 > \alpha$, (8) is modified as

$$O(\lambda_1, \lambda_2, \lambda_3) = \sum_{n=0}^{\infty} \sum_{p=0}^{2n} o_n^p I_n^p(\lambda_1) E_n^p(\lambda_1) E_n^p(\lambda_2) E_n^p(\lambda_3) \quad (10)$$

where

$$I_n^p(\lambda) = \int_{\lambda}^{\infty} \frac{dt}{E_n^p(t) \sqrt{(t^2 - h^2)(t^2 - k^2)}}. \quad (11)$$

$$E_n^p(\lambda) = \begin{cases} K(\lambda) = \lambda^{n-2r} \sum_{j=0}^r a_j \lambda^{2j} & p = [0, r] \\ L(\lambda) = \lambda^{1-n+2r} \sqrt{|\lambda^2 - h^2|} \sum_{j=0}^{r-1} a_j \lambda^{2j} & p = [r+1, n] \\ M(\lambda) = \lambda^{1-n+2r} \sqrt{|\lambda^2 - k^2|} \sum_{j=0}^{r-1} a_j \lambda^{2j} & p = [n+1, 2n-r] \\ N(\lambda) = \lambda^{n-2r} \sqrt{|\lambda^2 - h^2|} \sqrt{|\lambda^2 - k^2|} \sum_{j=0}^{r-2} a_j \lambda^{2j} & p = [2n-r+1, 2n] \end{cases} \quad (6)$$

$$\begin{aligned} K(\lambda) &= \lambda^{n-2r} \sum_{j=0}^r b_j \left(1 - \frac{\lambda^2}{h^2}\right)^j & p = [0, r] \\ L(\lambda) &= \lambda^{1-n+2r} \sqrt{|\lambda^2 - h^2|} \sum_{j=0}^r b_j \left(1 - \frac{\lambda^2}{h^2}\right)^j & p = [r+1, n] \\ M(\lambda) &= \lambda^{1-n+2r} \sqrt{|\lambda^2 - k^2|} \sum_{j=0}^r b_j \left(1 - \frac{\lambda^2}{h^2}\right)^j & p = [n+1, 2n-r] \\ N(\lambda) &= \lambda^{n-2r} \sqrt{|\lambda^2 - h^2|} \sqrt{|\lambda^2 - k^2|} \sum_{j=0}^r b_j \left(1 - \frac{\lambda^2}{h^2}\right)^j & p = [2n-r+1, 2n] \end{aligned} \quad (7)$$

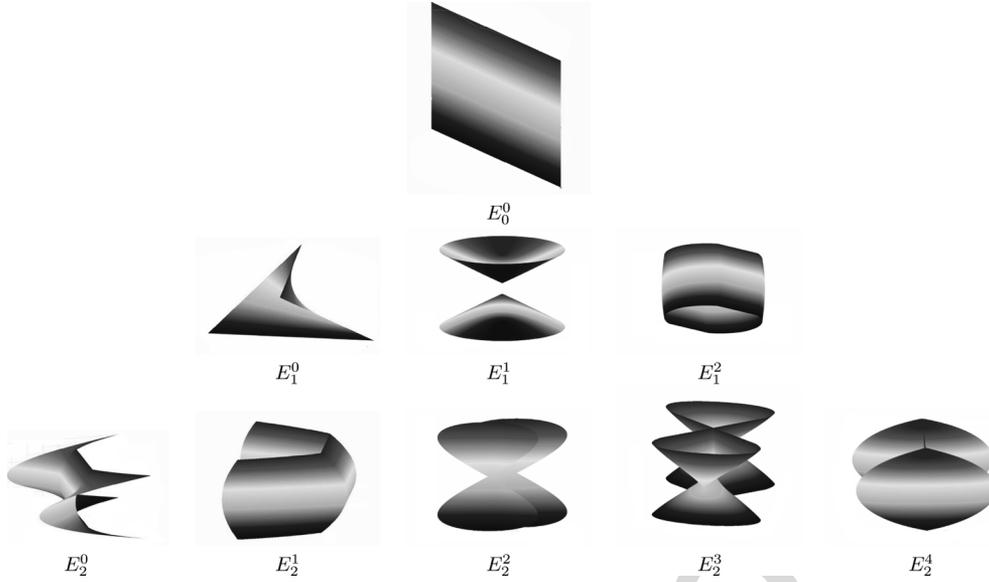


Fig. 2. Ellipsoidal harmonic expansion (in ellipsoidal coordinates).

The presence of $I_n^p(\cdot)$ ensures the stability of the result for $\lambda_1 > \alpha$, because

$$\lim_{\lambda \rightarrow \infty} E_n^p(\lambda) = \infty.$$

Fig. 2 depicts the first nine basis functions of ellipsoidal harmonics in the non-Euclidian space defined by ellipsoidal harmonics. In fact, every 3-D function is decomposed into a set of surfaces derived from the general three-variable quadratic equation (the curious reader may observe that E_1^1 is a two-sheet hyperboloid, E_2^0 is a saddle surface, and so on).

III. 3-D OBJECT DESCRIPTION USING ELLIPSOIDAL HARMONICS

In this paper, the use of the theory of ellipsoidal harmonics for 3-D object representation is proposed. The extracted ellipsoidal harmonics descriptors can then be utilized for 3-D search and retrieval applications. The major advantages of the ellipsoidal harmonics are identified as follows.

- Ellipsoids are better approximations for the majority of the 3-D objects. Spheres are the isotropic special cases of the ellipsoids and, thus, the approximation errors in some models are relatively large [33]. In contrast, an ellipsoidal implicit surface can alter its aspect ratio so as to fit better in a given model, resulting in reduced approximation errors. Thus, ellipsoidal harmonics are expected to result in a set of descriptors which has higher discriminative power when compared to similar approaches (e.g., spheres).
- Ellipsoidal harmonics result in a compact 3-D shape representation. The experiments performed proved that only few descriptors are required for accurate 3-D object representation.
- Ellipsoidal harmonics can be applied to 3-D objects that are expressed either on a surface representation or in a volumetric representation; thus, any error carried during the creation of an object can be avoided.

The key point in the extraction of the ellipsoidal harmonic descriptor is the appropriate selection of the function $O(\cdot)$ (8) which will be transformed according to the approach presented in Section II. In this virtue, three different approaches for the application of ellipsoidal harmonics are proposed: The surface ellipsoidal harmonics descriptor, which is applied in 3-D objects represented by surfaces, the volumetric ellipsoidal harmonics descriptor (VEHD), which is applied in the 3-D objects represented as volumetric functions and the generalized ellipsoidal harmonic descriptor (GEHD), applicable to any local feature of the 3-D object.

A. Ellipsoidal Harmonics Descriptor for Surface Represented 3-D Objects

The surface ellipsoidal harmonics descriptor (SEHD) can be computed as follows.

Firstly, the minimum bounding ellipsoid of the 3-D object is estimated and is considered as the reference ellipsoid of the object, i.e., the nine parameters of an ellipsoid are estimated:

$$\frac{x'^2}{a^2} + \frac{y'^2}{b^2} + \frac{z'^2}{c^2} = 1 \quad (12)$$

where

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R} & \mathbf{v} \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} \quad (13)$$

and the nine parameters are: a , b , and c which define the size of each axis of the ellipsoid, the three Euler angles represented by the 3×3 rotation matrix \mathbf{R} , and the three parameters that represent the absolute position of the ellipsoid's center in the global coordinate system (3×1 translation vector \mathbf{v}). The criterion is

the estimation of the bounding ellipsoid with minimum volume, i.e.,

$$V_{ell} = \frac{4}{3}\pi abc \rightarrow \min \quad (14)$$

and

$$\frac{x_i'^2}{a^2} + \frac{y_i'^2}{b^2} + \frac{z_i'^2}{c^2} \leq 1 \quad (15)$$

where (x_i, y_i, z_i) , $i = 1, \dots, N$ are the points of the 3-D object and V_{ell} is the ellipsoid's volume.

A very important result of this approach is the automatic estimation of the scaling, absolute, and relative position of the object in the 3-D space. Thus, a preprocessing step for normalization with respect to rotation, scaling, and translation is not required, as it is performed automatically during the minimization of (14).

Then, the surface representation is transformed so as to be expressed in the ellipsoidal coordinates. In the sequel, assuming that the input function $O(\cdot)$ in ellipsoidal harmonics analysis is $O(\lambda_1, \lambda_2, \lambda_3) = \lambda_1$, the descriptors o_n^p are computed according to (9).

It should be stated here that estimation of \mathbf{R} and \mathbf{v} can be also performed using principal component analysis of low order geometric moments. However, the resulting parameters a, b, c do not fulfill the criterion of (15). By definition, when PCA is utilized the estimated ellipsoidal parameters define the best fitted ellipsoid, which is different by the minimum volume bounding ellipsoid (Fig. 3).

B. Ellipsoidal Harmonics Descriptor for Volumetric 3-D Objects

The VEHD can be computed as follows.

Firstly, the minimum bounding ellipsoid of the 3-D object is estimated and is considered as the reference ellipsoid of the object. The estimation procedure is the same with the procedure of SEHD. Based on the reference ellipsoid, N ellipsoids are assumed at different scales and the intersection of the volume with every ellipsoid is considered. Then, every intersection is transformed so as to be expressed in the ellipsoidal coordinates and form a binary function $O(\cdot)$ in the curvilinear space of ellipsoidal coordinates. Finally, the descriptors o_n^p are computed according to (9).



Fig. 3. (a) Bounding ellipsoid (in purple) using the proposed analysis and (b) the best-fitted ellipsoid (in green) using PCA of the same object.

C. Minimum Bounding Ellipsoid of the 3-D Object

The minimum bounding ellipsoid of every object is estimated using the Computational Geometry Algorithms Library [34]. This computation estimates the absolute position of the object \mathbf{v} , the rotation matrix \mathbf{R} , and the parameters a, b, c of the object's reference ellipsoid. By doing so, normalization with respect to rotation and translation is automatically achieved, and thus, the rest of the process is invariant to rotation and translation. Moreover, using relative coordinates to the size of the ellipsoid, scaling invariance can also be achieved.

D. Computation of o_n^p Coefficients

Although the computation of o_n^p coefficient seems trivial and rather straightforward, the handling of the coordinates in a curvilinear space is rather tricky.

More specifically, a major problem during the computation of o_n^p is identified in the fact that the mapping between the Cartesian and ellipsoidal coordinates is not a one-to-one procedure. The initial solution for this problem is presented in [32], which extends the ellipsoidal coordinates $(\lambda_1, \lambda_2, \lambda_3)$ to the normalized ellipsoidal coordinates α, β, γ . However, this solution involves the computation of complete and incomplete 1st kind elliptic integrals and elliptic functions $sn(\cdot)$, $cn(\cdot)$, and $dn(\cdot)$ many times during the integration, which is time-consuming. As it is stated in [30], although the signs of λ_i are not known, the ambiguity which is being introduced can be easily overcome using the following notation:

$$E_n^p(\lambda_2, \lambda_3) = \Psi_n^p(\lambda_2, \lambda_3, x, y, z) P_n^p(\lambda_2) P_n^p(\lambda_3) \quad (16)$$

where

$$P_n^p(\lambda_i) = \sum_{j=0}^m b_j \left(1 - \frac{\lambda_i^2}{h^2}\right) \quad (17)$$

and $\Psi_n^p(\lambda_1, \lambda_2, \lambda_3, x, y, z)$ is given by (18) at the bottom of the page.

$$\Psi_n^p(\lambda_2, \lambda_3, x, y, z) = \begin{cases} (h k x)^{n-2r} & 0 \leq p \leq n-r \quad (K \text{ family}) \\ (h k x)^{1-(n-2r)} h y \sqrt{k^2 - h^2} & n-r+1 \leq p \leq n \quad (L \text{ family}) \\ (h k x)^{1-(n-2r)} k z \sqrt{k^2 - h^2} & n+1 \leq p \leq 2n-r \quad (M \text{ family}) \\ (h k x)^{n-2r} h k (k^2 - h^2) y z & 2n-r+1 \leq p \leq 2n \quad (N \text{ family}) \end{cases} \quad (18)$$

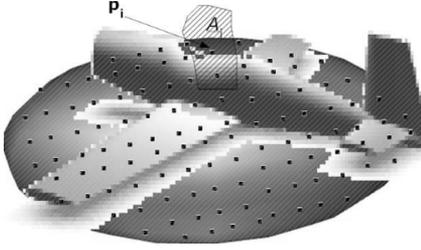


Fig. 4. Basic principle of the GEHD.

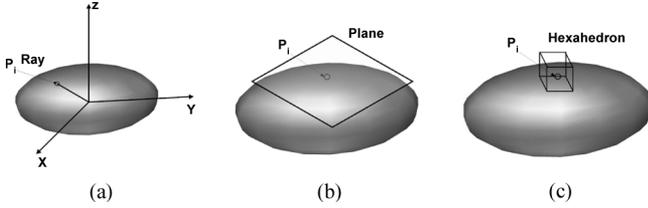


Fig. 5. Three instances of the generalized ellipsoidal harmonics using (a) a ray, (b) a tangent plane, and (c) a hexahedron.

Using (16) the computation of o_n^p coefficients overcomes the ambiguity of the non one-to-one mapping between the ellipsoidal and the Cartesian coordinates:

$$o_n^p = \int \int_S O(\lambda_1, \lambda_2, \lambda_3) \Psi_n^p(\lambda_2, \lambda_3, x, y, z) \times P_n^p(\lambda_2) P_n^p(\lambda_3) dS. \quad (19)$$

E. Generalized Ellipsoidal Harmonic Descriptor for any Kind of 3-D Object

The GEHD is the extension of SEHD and VEHD, so as the resulting descriptor captures more intuitive information.

For the computation of GEHD, firstly, N ellipsoids $E_k, k = 1, \dots, N$, confocal to the minimum bounding ellipsoid are defined and every ellipsoid E_k is sampled producing a set of sample points $P = \{\mathbf{p}_i, i = 1, \dots, S\}$. For each sample point \mathbf{p}_i , an area A_i centered in point \mathbf{p}_i is considered (Fig. 4). The part of the initial 3-D object which is included in the area A_i is analyzed using one or more local or global features (such as Fourier Descriptor, Zernike Descriptor [17], Wavelets, and Generalized Radon Transform [16]).

Let us assume that for the 3-D object Q , Q_{A_i} is the part of the 3-D object that lies in A_i . Q_{A_i} is analyzed using L different single-valued features $F_j(Q_{A_i}), j = 1, \dots, L$ (if a feature is multi-valued, then each value is a separate feature). Every function $F_j(Q_{A_i})$ can be considered as $F_j(\mathbf{p}_i)$, which can be analyzed using the ellipsoidal harmonics procedure according to (9) leading to descriptors $o_n^p(k, j)$. Fig. 4 depicts the sampling process and the selection of a single area A_i .

It should be noted that there is no limitation in the selection of the area A_i , except for $A_i = A_j \forall i \neq j$. A sphere, a box, and a N -hedron centered in \mathbf{p}_i [Fig. 5(c)], a plane tangent to \mathbf{p}_i [Fig. 5(b)], or a ray from the center of the ellipsoid that includes \mathbf{p}_i [Fig. 5(a)] are only few indicative types of the areas A_i that can be used. Furthermore, there is no limitation in the selection of the features computed in every Q_{A_i} .

A curious reader may notice that both VEHD and SEHD are special cases of the generalized ellipsoidal harmonic descriptor, where the functionals are the volume density and the distance from the center of the ellipsoid for VEHD and SEHD, respectively.

For the needs of this paper, the area A_i has been selected to be a cubic box, sized $5 \times 5 \times 5$ voxels, centered at \mathbf{p}_i . The selected functionals are appropriately selected coefficients of the 3-D wavelet transform, using Daubechies filters.

F. Example of Ellipsoidal Harmonics Descriptor Extraction

Let us assume that M is the 3-D object presented in Fig. 6. In this section, the process of extracting the VEHD will be thoroughly presented. The other two instances of ellipsoidal harmonic descriptors (SEHD and VEHD) are extracted in a similar manner.

The VEHD extraction process of M (Fig. 6, top-left) is as follows.

- The minimum bounding ellipsoid of the 3-D object is estimated according to [34] and is considered as the reference ellipsoid (Fig. 6, top-right).
- Then, 24 ellipsoids of different scales are defined and their intersections with the 3-D object are extracted. Fig. 6 (second line) depicts two intersections as examples.
- The i th intersection ($i = 1, \dots, N$) is a function defined on the surface of an ellipsoid. (Fig. 6, second line, presents two indicative intersections). It is transformed to the non-Euclidean space of ellipsoidal coordinates by solving (2) for every 3-D point forming a function $O_i(\cdot)$.
- Functions $O_i(\cdot)$ ($i = 1, \dots, N$) are analyzed in their ellipsoidal harmonics coefficients (Fig. 6, last line), resulting in the descriptor vector.

In the last line of the example presented in Fig. 6 is depicted the differences in the ellipsoidal harmonic expansion of two different functions. In cases where the expanded function does not present variations, the significant coefficients are few. The significant coefficients are more, when the function presents significant variations.

IV. COMBINED SPHERICAL—ELLIPSOIDAL HARMONIC DESCRIPTION

An innovative combination of spherical and ellipsoidal harmonics can be produced by performing multiple computation of ellipsoidal harmonics by placing the bounding ellipsoid in various orientations θ, ϕ of spherical coordinates and computing the spherical harmonic coefficients of every ellipsoidal coefficient (Fig. 7). The reference ellipsoid is rotated and ellipsoidal harmonics are computed at various orientations. Then, for every o_n^p coefficient, the spherical harmonics transformation is performed. The basic motivation behind this combination is to produce a novel 3-D feature that inherits both the “directional” information captured by ellipsoidal harmonics and the rotation invariant properties of spherical harmonics.

Specifically, the descriptor extraction procedure is the following: Initially, the nine parameters of the minimum bounding ellipsoid are estimated, according to the methodology presented in the previous section. The ellipsoid is placed in the appropriate position; however, rotation normalization is not

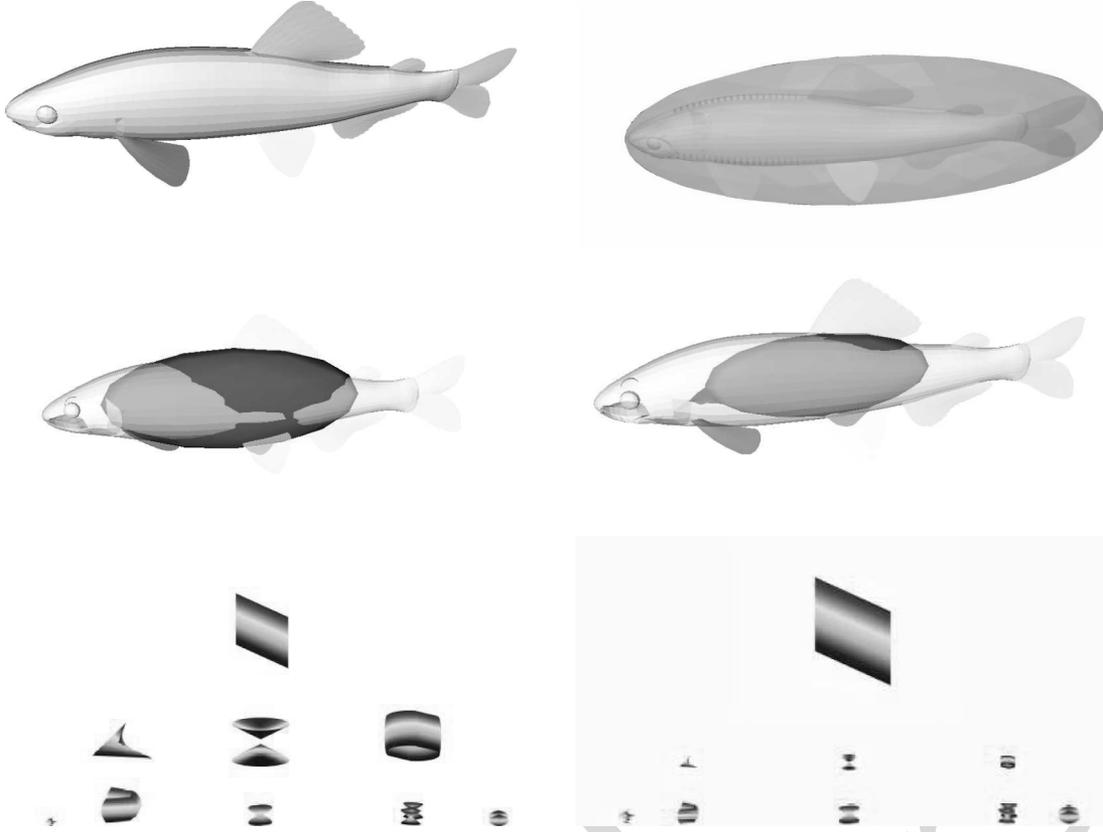


Fig. 6. Extraction process: The minimum bounding ellipsoid of the initial object (top left) is estimated (top right). A function defined in an ellipsoid is defined, in this example two intersections of the ellipsoid with 3-D object are presented (second line). The dark red areas of the ellipsoid are valued with zero and the light areas are valued with a non-zero value. These function is analyzed according to ellipsoidal harmonics basis functions as can be seen in the last line.

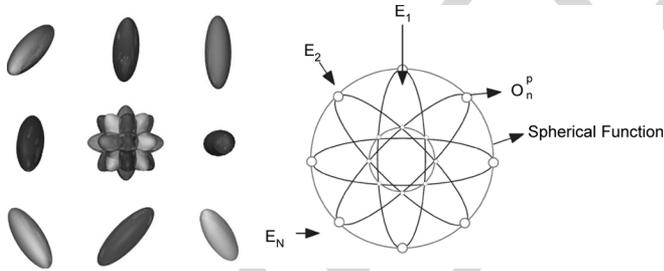


Fig. 7. Combining spherical and ellipsoidal harmonics.

performed. In the sequel, various orientations of the reference ellipsoid are considered. For each orientation, ellipsoidal harmonic descriptors are computed, using either VEHD, GEHD, or SEHD (Fig. 7). Every ellipsoidal harmonic descriptor from all orientations forms a spherical function defined in θ, ϕ of spherical coordinates. Finally, every spherical function is analyzed according to spherical harmonics analysis, and the resulting coefficients are forming the descriptor vector of the 3-D object, the combined spherical and ellipsoidal harmonics descriptor (SH + EHD).

It should be noted that every ellipsoidal harmonic descriptor can be extended to the combined spherical-ellipsoidal harmonics descriptor. The major advantage of the latter approach

is the computation of native rotation invariant descriptors, which overcome the errors posed from incorrect rotation estimation of the initial 3-D object.

Lemma: Combined Spherical-Ellipsoidal Harmonic Description Is Rotation Invariant:

Proof: Let O be a 3-D object and $o_p^n(\theta, \phi)$ its ellipsoidal harmonic descriptors, when the reference ellipsoid major axis is placed on θ, ϕ of spherical coordinates.

Let also O^R be the same 3-D object rotated around an arbitrary axis by R and ${}^R o_p^n(\theta, \phi)$ its ellipsoidal harmonic descriptors. It is obvious that $\forall(\theta, \phi) \exists(\theta_R, \phi_R)$ which is the result of the rotating the axis (θ, ϕ) around the same axis by R , where $o_p^n(\theta, \phi) = {}^R o_p^n(\theta_R, \phi_R)$ and that $o_p^n(\theta + \delta\theta, \phi + \delta\phi) = {}^R o_p^n(\theta_R + \delta\theta, \phi_R + \delta\phi)$. In fact, the function ${}^R o_p^n(\cdot)$ is a rotated version of $o_p^n(\cdot)$.

Let $T[\cdot]$ be the transformation of a spherical function to its rotation invariant spherical harmonic expansion. Then, $T[{}^R o_p^n(\cdot)] = T[o_p^n(\cdot)]$.

V. MATCHING METHOD

Let us assume that two 3-D objects O_1 and O_2 are described using the ellipsoid harmonics descriptors $o_n^{p(1)}$ and $o_n^{p(2)}$, respectively. In order to calculate a similarity metric between the

two objects, the normalized Minkowski L_1 distance has been utilized:

$$L_1(O_1, O_2) = \sum_{n=0}^{n=N_h-1} \sum_{p=0}^{p=2n} \left| o_n^{p(1)} - o_n^{p(2)} \right| \quad (20)$$

where

$$o_n^{p(i)} = \frac{o_n^{p(i)}}{\sum_{n=0}^{n=N_h-1} \sum_{p=0}^{p=2n} o_n^{p(i)}} \quad (21)$$

and N_h is the maximum order of harmonics.

VI. COMPUTATIONAL ASPECTS

The computational cost of the SEHD and VEHD methods can be analyzed as follows.

1) *Minimum Bounding Ellipsoid Estimation*: This part of the procedure is crucial, because it defines the basis of the curvilinear coordinate system. This computation is performed only once and according to the CGAL [34], its complexity is $O(9k(\epsilon^{-1} + \ln 3 + \ln \ln k))$, where k is the number of model points. In practice, this part of the procedure is completed in less than 0.01 s for a typical 3-D object in the testing machine.

2) *Computation of o_n^p Coefficients*: The computation of the coefficients depends on two factors.

- The integration algorithm: For the integration, the traditional brute-force approach has been followed (i.e., summation of the values in every sample). Thus, a smart sampling approach has been adopted, where $S = 5200$ samples are uniformly selected over the surface of the ellipsoid for VEHD and SEHD. Using more sample points, the computational time is increased, while there is no serious affect in the retrieval performance of the approach. For the GEHD, the sampling rate is $S = 1300$ samples per ellipsoid, which is smaller due to the fact that GEHD is based on an area around the sample point, resulting in local descriptors that comprise information of the surrounding area of the sample.
- The algorithm for polynomial $P_n^p(\cdot)$ computation: one of the most efficient algorithms has been adopted for the needs of this paper, the simple Horner's rule that requires exactly n multiplications and n additions for every value of $P_n^p(\cdot)$.

The computational complexity of the proposed approach SH + EHD is due to:

- computation of lame polynomials (once);
- computation of ellipsoidal harmonics coefficients for all possible directions of the ellipsoid;
- computation of spherical harmonics (once).

A major advantage of the SH + EHD is identified by the fact that the computation of ellipsoidal harmonics coefficients for all possible directions is a procedure which can be easily performed using multithreaded processing.

TABLE III
COMPUTATION TIMES FOR THE VARIOUS INSTANCES
OF ELLIPSOIDAL HARMONIC DESCRIPTOR

Method	Mean Computation Time
SEHD	1.02 secs
VEHD	1.57 secs
GEHD	1.95 secs
SH+EHD	6.37 secs
REXT	0.58 secs
GEDT	2.4 secs
LFD	2.77 secs

VII. EXPERIMENTAL RESULTS

The proposed approach has been evaluated for its retrieval performance using the Princeton Shape Benchmark (PSB) [35], the Engineering Shape Benchmark (ESB) [36] and the ITI 3-D model's database [16] (the ITI database can be downloaded from <http://www.victory-eu.org>) and has been compared to well-known approaches of Gaussian Euclidean distance transform (GEDT), which is based on the comparison of a 3-D function, whose value at each point is given by composition of a Gaussian with the Euclidean distance transform of the surface [9], the Light field descriptor (LFD) where a representation of a model as a collection of images rendered from uniformly sampled positions on a view sphere is utilized [11], and the Radialized spherical extent function (REXT) where a collection of spherical functions giving the maximal distance from center of mass as a function of spherical angle and radius is utilized [12]. The retrieval accuracy has been evaluated using the precision-recall diagrams, where precision is defined as the ratio of the relevant retrieved objects and the total number of the retrieved objects, and recall is the ration of the relevant retrieved objects and the total relevant objects in the database. For the ESB and PSB datasets, which adopt multilevel classification schemes, an object is considered similar to the query if both objects belong to the same subclass.

The experiments performed using an Intel Core Duo running at 1.5 GHz per core with 2 GB RAM, running Windows XP. The source code has been compiled using Microsoft Visual Studio 8.0 SP1, utilizing multithreaded processing techniques. The results concerning the approaches of [9], [11], and [12] have been derived using the executables provided by the authors of corresponding articles.

The volumetric ellipsoidal harmonic descriptor and the generalized ellipsoidal harmonic descriptor have been computed for 3-D objects that are expressed in a $64 \times 64 \times 64$ cubic voxel grid. The number of the ellipsoids has been selected to be $N = 24$ (for VEHD) and $N = 12$ (for GEHD) and the maximum degree of expansion $N_h = 6$. The combined spherical-ellipsoidal descriptor (SH + EHD) has been computed for $N = 5$ ellipsoids and 342 different orientations.

Firstly, the computational complexity of the proposed approach has been evaluated in terms of execution time. Table III presents the average computational time required for the computation for the various instances of the ellipsoidal harmonics descriptors and the methods of [9], [11], and [12].

The proposed methods require slightly more execution time than the methods presented in the literature. However, the latter cannot be considered as a major drawback because the execution

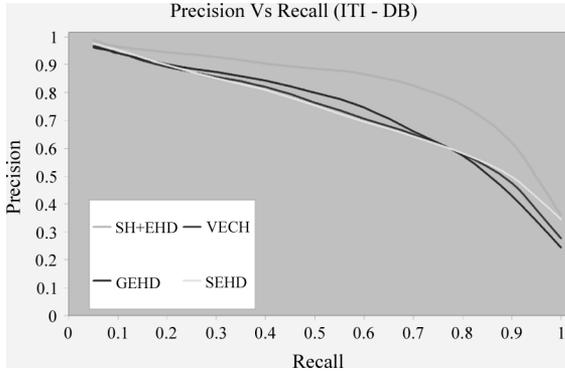


Fig. 8. Comparative precision-recall diagrams for the various instances of EH in the ITI database.

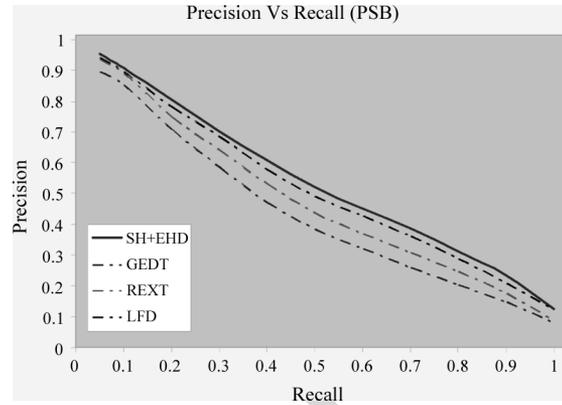


Fig. 10. Comparative precision-recall diagrams for the Princeton Shape Benchmark.

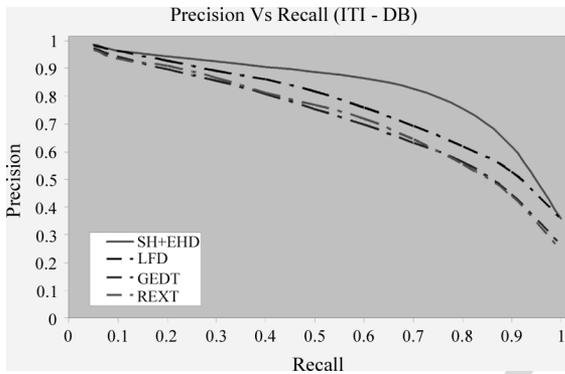


Fig. 9. Comparative precision-recall diagrams for the ITI database.

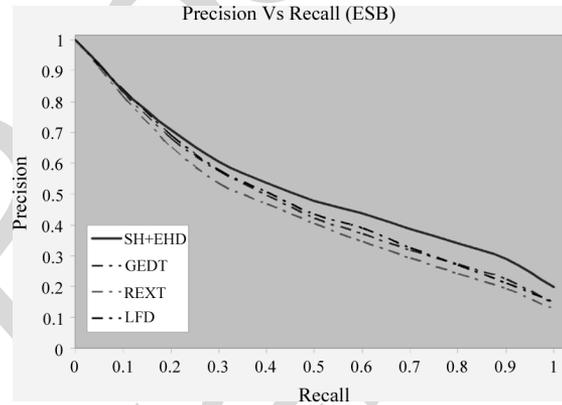


Fig. 11. Comparative precision-recall diagrams for the Engineering Shape Benchmark.

times are small enough to be used in practical search problems, and the hardware is continuously improved leading in smaller execution times. Moreover, the descriptor extraction takes place only once for each 3-D object and the descriptors are stored in the database. During retrieval, the descriptors are retrieved from the database and are compared to the query's descriptors.

In the sequel, the rotation invariance of SH + EHD descriptor is experimentally verified. A set of sample objects has been arbitrarily rotated and the SH + EHD descriptors for both original and rotated sets have been computed. The maximum difference between the original and rotated version of the same object was 0.5%. The small differences are expected in practice due to the discrimination of the continuous spherical harmonic process.

Fig. 8 depicts the comparative performance of the instances of the ellipsoidal harmonic descriptor on the ITI database. It is observed that GEHD's retrieval accuracy outperforms VEHD's and SEHD which are comparable. This can be explained by the fact that both VEHD and SEHD are trivial instances of GEHD. When another kind of information is considered (e.g., the 3-D wavelet transform utilized in this paper), GEHD provides slightly better results. It is also obvious that the combined spherical-ellipsoidal harmonic descriptor outperforms the simple ellipsoidal and the generalized ellipsoidal harmonic descriptors and, thus, is selected to be compared to the other approaches. Figs. 9–11 depict the comparative performance of the best instance (SH+EHD) of the ellipsoidal harmonic descriptor

for the three different databases compared to the approaches of [9], [11], and [12]

By comparing the precision-recall diagrams for the three different databases, it is obvious that all the approaches have completely different behavior in different databases. The latter basically happened due to the different nature of each database, the number of the 3-D objects it contains, and the way that the 3-D objects have been classified.

The efficiency of ellipsoidal harmonics is depicted in the combined spherical-ellipsoidal harmonic descriptor, which outperforms all competitive approaches and any other ellipsoidal harmonic descriptor. The combined spherical-ellipsoidal harmonic descriptor combines the rotation invariant features provided by the spherical harmonic analysis, with directional features captured by the ellipsoidal harmonic analysis, and thus, the resulting descriptor has significantly better retrieval performance when compared to other approaches.

It is very interesting that the performance of the proposed approach is significantly greater than the other approaches for Recall > 0.4 in the CAD database (Fig. 11) and in the ITI database (Fig. 9). The latter means that all relevant objects will be presented earlier to the user when compared to other approaches. Both ITI and ESB are mainly composed of objects

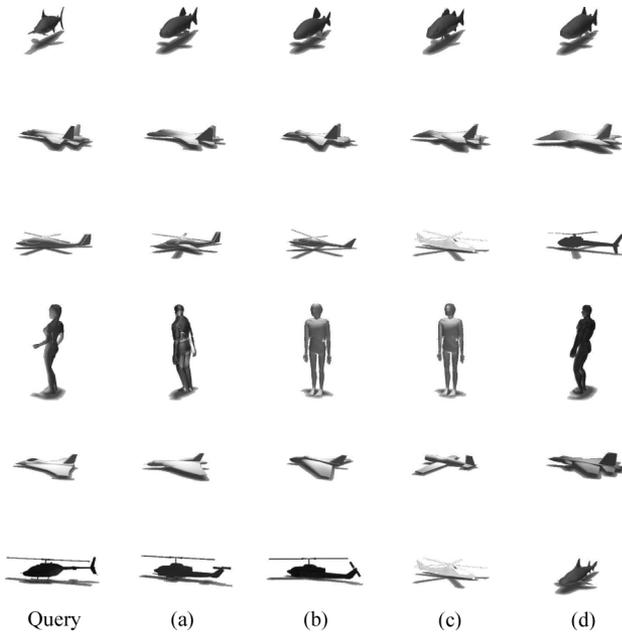


Fig. 12. Indicative retrieved results. The first column depicts the query 3-D object while the rest are the first four retrieved objects.

having mainly directional information, and thus, the combination of the rotation invariant features provided by the spherical harmonic analysis, with directional features captured by the ellipsoidal harmonic analysis, achieve better retrieval efficiency in these databases.

In Fig. 12, selected retrieved results are shown. The first column is the query model, while the rest are the first four retrieved results. From the presented results, the geometric similarity between the query and the retrieved objects is obvious; however, there are semantic differences in some retrieved results, e.g., in the last two lines of Fig. 12. When the query is a “delta plane”, a conventional plane is ranked third, and when the query is a helicopter, a fish is ranked in the fourth position. Results of this kind are expected, since neither topology nor high-level information has been taken into account that could semantically discriminate these 3-D objects. From a geometric perspective, the major difference between a helicopter and a fish is located in the helicopter’s wings (the different scaling has been discarded during normalization). In the proposed approach, intersections of the 3-D object with many ellipsoids are utilized to extract the shape descriptors. When comparing the intersections of a helicopter to the intersections of a fish, only few intersections differ significantly, while a great majority is highly similar, and thus, the mean dissimilarity score will be low. Additionally, two objects of the same class may be normalized in slightly different scale, and thus, few intersections will present significantly high dissimilarity. As a result, scores between some helicopters can potentially be comparable to the scores between a helicopter and a fish, resulting in retrievals similar to the one presented in the last line of Fig. 13, where a fish is ranked fourth when the query is a helicopter.

VIII. CONCLUSIONS

In this paper, various novel descriptors based on ellipsoidal harmonics were introduced. The combination of spherical and ellipsoidal harmonics results in a more discriminative descriptor set which is capable of performing robust 3-D content-based search and retrieval for online applications. The experimental results proved the efficiency of the proposed descriptors in performing geometry-based 3-D object search and retrieval. Although geometry-based content retrieval provides very good results, the geometry of a 3-D object may not always provide the semantically similar results. In these cases, the geometry-based results should be combined in a semantic-based framework where the system is enhanced with external knowledge in order to improve the retrieval performance.

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Athanasios Mademlis was born in Thessaloniki, Greece, in 1980. He received the Diploma degree in electrical and computer engineering and the Ph.D. degree in electrical and computer engineering from Aristotle University of Thessaloniki, Thessaloniki, Greece, in 2004 and 2009, respectively.

Dr. Mademlis is a member of the Technical Chamber of Greece.

Petros Daras (M'XX) **AUTHOR: WHAT YEAR?** was born in Athens, Greece, in 1974. He received the Diploma degree in electrical and computer engineering, the M.Sc. degree in medical informatics, and the Ph.D. degree in electrical and computer engineering from the Aristotle University of Thessaloniki, Thessaloniki, Greece, in 1999, 2002, and 2005, respectively.

He is a Senior Researcher at the Informatics and Telematics Institute. His main research interests include computer vision, search and retrieval of 3-D objects, and multimedia search engines. His involvement with those research areas has led to the coauthoring of more than 12 papers in refereed journals and more than 40 papers in international conferences. He has served as a regular reviewer for a number of international journals and conferences. He has been involved in more than 15 European and National research projects.

Dr. Daras is a member of the Technical Chamber of Greece.

Dimitrios Tzovaras received the Diploma degree in electrical engineering and the Ph.D. degree in 2-D and 3-D image compression from Aristotle University of Thessaloniki, Thessaloniki, Greece, in 1992 and 1997, respectively.

He is a Senior Researcher in the Informatics and Telematics Institute of Thessaloniki. Prior to his current position, he was a Senior Researcher on 3-D imaging at the Aristotle University of Thessaloniki. His main research interests include virtual reality, assistive technologies, 3-D data processing, medical image communication, 3-D motion estimation, and stereo and multiview image sequence coding. His involvement with those research areas has led to the coauthoring of more than 35 papers in refereed journals and more than 80 papers in international conferences. He has served as a regular reviewer for a number of international journals and conferences. Since 1992, he has been involved in more than 40 projects in Greece, funded by the EC, and the Greek Secretariat of Research and Technology.

Dr. Tzovaras is a member of the Technical Chamber of Greece.

Michael Gerassimos Strintzis (M'70–SM'80–F'04) received the Diploma degree in electrical engineering from the National Technical University of Athens, Athens, Greece, in 1967, and the M.A. and Ph.D. degrees in electrical engineering from Princeton University, Princeton, NJ, in 1969 and 1970, respectively.

He then joined the Electrical Engineering Department at the University of Pittsburgh, Pittsburgh, PA, where he served as Assistant Professor (1970–1976) and Associate Professor (1976–1980). Since 1980, he has been a Professor of electrical and computer engineering at the University of Thessaloniki, Thessaloniki, Greece, and, since 1999, Director of the Informatics and Telematics Research Institute, Thessaloniki. His current research interests include 2-D and 3-D image coding, image processing, biomedical signal and image processing, and DVD and Internet data authentication and copy protection.

Dr. Strintzis has served as an Associate Editor for the IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS FOR VIDEO TECHNOLOGY since 1999. In 1984, he was awarded one of the Centennial Medals of the IEEE.