ACKNOWLEDGMENT

This work was supported by the EC under contracts FP7-248984 GLOCAL and FP7-287911 LinkedTV.

APPENDIX A
DERIVATION OF EQUATIONS IN SECTION II

A. Derivation of Eqs. (6) and (7)

The Gaussian mixture distribution concerning the i-th class in (5) can be derived in terms of latent variables [36], [37], as described in the following. Let $Z_i \in \mathbb{R}^{H_i}$ be a categorical latent random vector concerning the i-th class, whose parameter space $\mathcal{Z}_i$ is the standard base of $\mathbb{R}^{H_i}$, i.e., $\mathcal{Z}_i = \{\mathbf{e}_{i,1}, \ldots, \mathbf{e}_{i,H_i}\}$, where only the j-th element of the unit vector $\mathbf{e}_{i,j}$ is equal to one and all other elements are equal to zero. Setting $p(Z_i = \mathbf{e}_{i,j}) = \pi_{i,j}$ and $p(\mathbf{x}|Z_i = \mathbf{e}_{i,j}) = \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_{i,j})$ the marginal and conditional densities, $p(\mathbf{z}_i)$ and $p(\mathbf{x}|\mathbf{z}_i)$, are expressed in terms of the mixing coefficients and mixture components respectively, $p(\mathbf{z}_i) = \prod_{j=1}^{H_i} \pi_{i,j}^{z_{i,j}}$, $p(\mathbf{x}|\mathbf{z}_i) = \prod_{j=1}^{H_i} \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_{i,j})^{z_{i,j}}$. Thus, using the product rule of probability we can express the i-th class-conditional joint density as

$$\begin{aligned} p(\mathbf{x}, \mathbf{z}_i|\omega_i) &= p(\mathbf{z}_i|\omega_i)p(\mathbf{x}|\mathbf{z}_i, \omega_i) = p(\mathbf{z}_i)p(\mathbf{x}|\mathbf{z}_i) \\ &= \prod_{j=1}^{H_i} (\pi_{i,j}\mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_{i,j}))^{z_{i,j}}, \end{aligned} \tag{52}$$

where we have used the fact that $\mathbf{x}$ is conditionally independent of ω_i given $\mathbf{z}_i$, and $\mathbf{z}_i$ is independent of ω_i. The i-th class-conditional marginal distribution of $\mathbf{x}$ can then be written as

$$p(\mathbf{x}|\omega_i) = \sum_{\mathbf{z}_i} p(\mathbf{x}, \mathbf{z}_i|\omega_i) = \sum_{j=1}^{H_i} \pi_{i,j}\mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_{i,j}), \tag{53}$$

which is a Gaussian mixture equivalent to (5), and, using the Bayes' rule the posterior distribution is also derived

$$p(\mathbf{z}_i|\mathbf{x}, \omega_i) = \frac{\prod_{j=1}^{H_i} (\pi_{i,j}\mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_{i,j}))^{z_{i,j}}}{\sum_{j=1}^{H_i} \pi_{i,j}\mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_{i,j})}. \tag{54}$$

Therefore, under the i.i.d. assumption, the likelihood of the complete data set is expressed as (p.108, [27])

$$\begin{aligned} p(\mathbf{X}, \mathbf{Z}|\boldsymbol{\theta}) &= \prod_{i=1}^{C} \prod_{n=1}^{N_i} p(\mathbf{x}_i^n, \mathbf{z}_i^n|\omega_i) \\ &= \prod_{i=1}^{C} \prod_{n=1}^{N_i} \prod_{j=1}^{H_i} (\pi_{i,j}\mathcal{N}(\mathbf{x}_i^n|\boldsymbol{\mu}_{i,j}))^{z_{i,j}^n}. \end{aligned} \tag{55}$$

while the posterior distribution takes the form

$$p(\mathbf{Z}|\mathbf{X}, \boldsymbol{\theta}) \propto \prod_{i=1}^{C} \prod_{n=1}^{N_i} \prod_{j=1}^{H_i} (\pi_{i,j}\mathcal{N}(\mathbf{x}_i^n|\boldsymbol{\mu}_{i,j}))^{z_{i,j}^n}, \tag{56}$$

where $\mathbf{Z} = \{\mathbf{Z}_1, \ldots, \mathbf{Z}_C\}$ is the set of all categorical vectors. Observing that the posterior distribution is independent over $z_{i,j}^n$, the expectation of the categorical variables can be derived

$$\mathbb{E}[z_{i,j}^n] = \frac{\sum_{j=1}^{H_i} z_{i,j}^n (\pi_{i,j}\mathcal{N}(\mathbf{x}_i^n|\boldsymbol{\mu}_{i,j}))^{z_{i,j}^n}}{\sum_{j=1}^{H_i} \pi_{i,j}\mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_{i,j})}, \tag{57}$$

and simplifying the above, we arrive to the definition of the responsibilities in (7).

Moreover, from (56) the log likelihood of the complete data set is retrieved

$$\ln p(\mathbf{X}, \mathbf{Z}|\boldsymbol{\theta}) = \sum_{i=1}^{C} \sum_{n=1}^{N_i} \sum_{j=1}^{H_i} z_{i,j}^n (\ln \pi_{i,j} + \ln \mathcal{N}(\mathbf{x}_i^n|\boldsymbol{\mu}_{i,j})). \tag{58}$$

Applying the expectation operator to the above expression and substituting $\mathbb{E}[z_{i,j,n}]$ from (7) the expectation of the complete data log-likelihood is expressed as

$$\begin{aligned} &\mathbb{E}[\ln p(\mathbf{X}, \mathbf{Z}|\boldsymbol{\theta})] \\ &= \sum_{i=1}^{C} \sum_{n=1}^{N_i} \sum_{j=1}^{H_i} h_{i,j}^n (\ln \pi_{i,j} + \ln \mathcal{N}(\mathbf{x}_{i,n}|\boldsymbol{\mu}_{i,j}, \boldsymbol{\Sigma})) \\ &= \sum_{i=1}^{C} \sum_{j=1}^{H_i} \tilde{N}_{i,j} \ln \pi_{i,j} - \frac{NF}{2} \ln(2\pi) + \frac{N}{2} \ln |\boldsymbol{\Sigma}^{-1}| \\ &- \frac{1}{2} \sum_{i=1}^{C} \sum_{n=1}^{N_i} \sum_{j=1}^{H_i} h_{i,j,n} (\mathbf{x}_{i,n} - \boldsymbol{\mu}_{i,j})^T \boldsymbol{\Sigma}^{-1} (\mathbf{x}_{i,n} - \boldsymbol{\mu}_{i,j}). \end{aligned} \tag{59}$$

Using the identity $(\mathbf{x}_i^n - \boldsymbol{\mu}_{i,j})^T \boldsymbol{\Sigma}^{-1} (\mathbf{x}_i^n - \boldsymbol{\mu}_{i,j}) = (\mathbf{x}_i^n - \bar{\mathbf{x}}_i^n)^T \boldsymbol{\Sigma}^{-1} (\mathbf{x}_i^n - \bar{\mathbf{x}}_{i,j}) + (\bar{\mathbf{x}}_{i,j} - \boldsymbol{\mu}_{i,j})^T \boldsymbol{\Sigma}^{-1} (\bar{\mathbf{x}}_{i,j} - \boldsymbol{\mu}_{i,j}) + 2(\mathbf{x}_i^n - \bar{\mathbf{x}}_{i,j})^T \boldsymbol{\Sigma}^{-1} (\bar{\mathbf{x}}_{i,j} - \boldsymbol{\mu}_{i,j})$ along with the fact that $\sum_{n=1}^{N_i} (\mathbf{x}_i^n - \bar{\mathbf{x}}_{i,j})^T \boldsymbol{\Sigma}^{-1} (\bar{\mathbf{x}}_{i,j} - \boldsymbol{\mu}_{i,j}) = 0$, and multiplying both sides by two, we arrive to (6).

B. Derivation of Eq. (18)

The constraint that the mixing coefficients should sum to one can be incorporated in (17) using C lagrange multipliers $\eta_i, i = 1, \ldots, C$. Therefore, we need to find the stationary point of

$$\begin{aligned} &\sum_{i=1}^{C} \sum_{n=1}^{N_i} \sum_{j=1}^{H_i} h_{i,j}^n (\ln \pi_{i,j} + \ln \mathcal{N}(\mathbf{x}_i^n|\boldsymbol{\mu}_{i,j})) \\ &+ \sum_{i=1}^{C} \eta_i (\sum_{j=1}^{H_i} \pi_{i,j} - 1) \end{aligned} \tag{60}$$

with respect to $\pi_{i,j}$ and η_i. Optimizing over $\pi_{i,j}$ we arrive to $\tilde{N}_{i,j}/\pi_{i,j} + \eta_i = 0$. If we multiply both sides with $\pi_{i,j}$ and sum over all subclasses of the i-th class we get $\eta_i = -N_i$. Eliminating η_i we obtain (18).

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