REGION-BASED ACTIVE CONTOURS USING GEOMETRICAL AND STATISTICAL FEATURES FOR IMAGE SEGMENTATION

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ABSTRACT

We consider the problem of image segmentation through the minimization of an energy criterion involving both region and boundary functionals. We study the derivation of these functionals using the notion of shape derivative. From the derivative, we deduce the evolution equation of an active contour that will make it evolve towards a minimum of the criterion introduced.

We focus on geometric and statistical features globally attached to the boundary or to the region, and we take explicitly into account their evolution in the derivation. First, statistical region-based descriptors using the variance of a region or the distance to a reference region histogram are introduced. Then a geometric prior term is combined with statistical features for homogeneous region segmentation. This geometric prior is introduced to provide a free form deformation from a reference shape. Some experimental results on real images and video sequences show the benefit of combining geometrical and statistical features for segmentation.

1. INTRODUCTION

In this paper, the segmentation problem is cast in a variational framework by introducing a functional to minimize following the pioneer work of Mumford and Shah [1]. A minimum of this functional is searched via the propagation of an active contour following the pioneer work of Kass et al. [2] and Caselles et al. [3].

In the two-dimensional case, the evolving boundary, or active contour, is modeled by a parametric curve \( \Gamma(s, \tau) \), where \( s \) may be its arc-length and \( \tau \) is an evolution parameter. The active contour is then driven by the following Partial Differential Equation (PDE):

\[
\frac{\partial \Gamma(s, \tau)}{\partial \tau} = v = FN \quad \text{with} \quad \Gamma(\tau = 0) = \Gamma_0,
\]

where \( \Gamma_0 \) is an initial curve defined by the user and \( v \) the velocity vector of \( \Gamma(s, \tau) \) of amplitude \( F \) directed along the inward normal vector \( N \) of \( \Gamma \). This velocity is the unknown that has to be derived from a functional so that the solution \( \Gamma(\cdot, \tau) \) converges towards a curve achieving a local minimum, and thus, hopefully, towards the boundary of the object to segment.

In order to construct the PDE that will drive the active contour towards a minimum of the functional, derivation of the functional is needed to characterize the minimum. We are interested more particularly in region-based active contours [4, 5, 6, 7, 8] where the functional is a combination of domain integrals and boundary integrals, see [9] for a detailed review of these methods. In this case, we search for an optimal domain and so we propose to derive the functional using shape derivation tools [10].

In section 2, we focus on the derivation of region-dependent features that evolve during the propagation of the curve. It happens when statistical descriptors, such as the variance of a region or the histogram of a region are used. We propose functions of the variance [9] for segmentation of homogeneous regions, and distance between histograms for matching purposes [11]. Experimental results using the distance between histograms are provided for matching regions in video sequences.

In section 3, we deal with boundary-dependent features such as geometrical descriptors using the distance to a reference curve [12]. Experimental results take benefit of the combination of such geometrical descriptors with statistical features for segmentation.

2. STATISTICAL REGION-BASED DESCRIPTORS

Let \( \mathcal{U} \) be a class of domains (open, regular bounded sets, i.e. \( C^2 \)) of \( \mathbb{R}^n \), and \( \Omega \) an element of \( \mathcal{U} \). The boundary \( \partial \Omega \) of \( \Omega \) is sometimes denoted by \( \Gamma \).

The region-based term is expressed as a domain integral
of a function $k_r$ named descriptor of the region:

$$J_r(\Omega) = \int_{\Omega} k_r(x, \Omega) dx$$  \hspace{1cm} (1)$$

In the general case this descriptor may depend on the domain such as the statistical descriptors introduced thereafter.

The derivation of this term is performed using domain derivation tools. We apply the following theorem which gives a relation between the Eulerian derivative of $J_r(\Omega)$ in the direction $\mathbf{V}$, and the domain derivative of $k_r$ denoted $k'_r(x, \Omega, \mathbf{V})$, see [11, 9] for details:

$$dJ_r(\Omega, \mathbf{V}) = \int_{\Omega} k'_r(x, \Omega, \mathbf{V}) dx - \int_{\partial\Omega} k_r(x, \Omega) (\mathbf{V} \cdot \mathbf{N}) ds$$

The first integral comes from the dependence of the descriptor $k_r(x, \Omega)$ upon the region while the second term comes from the evolution of the region itself.

In order to explicitly compute the evolution equation in the case of region-dependent descriptors, the first integral has to be evaluated. We propose here two main examples of region-dependent descriptors and we compute the associated evolution equation. In these examples, the general form of the evolution equation is the following:

$$\frac{\partial \Gamma}{\partial t} = \left[ k_r(x, \Omega) + A(x, \Omega) \right] \mathbf{N}$$  \hspace{1cm} (2)$$

where $x = \Gamma$ in this equation, and $A(x, \Omega)$ is a term coming from the dependence of the descriptors with the region, computed through the evaluation of the domain integral of the domain derivative $k'_r$. We call this term “region-dependent term” and we propose its computation for two examples thereafter.

2.1. Statistical descriptors based on the variance

Given the image intensity $I : \Omega_I \rightarrow \mathcal{R}$ with $\Omega_I \subset \mathcal{R}^n$, we may intend to minimize the intensity variance of a region, $\sigma^2(\Omega)$, for homogeneous regions segmentation in greyscale images. We minimize functional (1) with respect to $\Omega$ with the following descriptor:

$$k_r(x, \Omega) = q(\sigma^2(\Omega))$$

where $q : \mathcal{R}^+ \rightarrow \mathcal{R}^+$ is a positive function of class $C^1$.

The region-dependent evolution equation is of the form (2). The region-dependent term coming from the dependence of the descriptor with the region is the following (see [9] for details on the computation):

$$A(x, \Omega) = q'(\sigma^2(\Omega))((I(x) - \mu(\Omega))^2 - \sigma^2(\Omega))$$  \hspace{1cm} (3)$$

This computation may also be extended to statistical descriptors modeled as a linear combination of region integrals such as the covariance matrix determinant, see [11, 9] for details.

2.2. Statistical descriptors based on histograms

Then, we compute the evolution equation from a functional including descriptors using the distance between the histogram of the region and a reference histogram for matching purposes [11]. Given the image intensity $I : \Omega_I \rightarrow \mathcal{R}^m$, we define $q(\alpha, \Omega)$ as the histogram of the region $\Omega$ and $q(\alpha) : \mathcal{R}^m \rightarrow [0, 1]$ as the histogram of the reference region $\Omega_{ref}$.

We minimize the distance between the reference histogram $q_{ref}(\alpha)$ and the region histogram $q(\alpha, \Omega)$ by minimizing functional (1) with respect to $\Omega$ using the following descriptor:

$$k_r(x, \Omega) = \int_{\mathcal{R}^m} \varphi(q(\alpha, \Omega), \alpha) d\alpha$$  \hspace{1cm} (4)$$

where $\varphi : \mathcal{R}^+ \times \mathcal{R}^m \rightarrow \mathcal{R}^+$ may be $\varphi(q(\alpha, \Omega), \alpha) = (\sqrt{q(\alpha, \Omega)} - \sqrt{q(\alpha)})^2$ for the Hellinger distance or $\varphi(q(\alpha, \Omega), \alpha) = (q(\alpha, \Omega) - q(\alpha))^2/q(\alpha)$ for the chi-2 comparison function. We can estimate the histogram $q(\alpha, \Omega)$ with the Parzen method [13]:

$$\hat{q}(\alpha, \Omega) = \frac{1}{K(\Omega)} \int_{\Omega} g_\sigma(I(x) - \alpha) dx$$

where the kernel is an $m$-dimensional Gaussian with 0-mean and variance $\sigma^2$ and with $K(\Omega) = |\Omega|$.

The resulting evolution equation is equation (2) where the region-dependent term is the following (see [11] for details):

$$A(x, \Omega) = -C(\Omega) + \partial_1 \varphi_{in}(\hat{q}(\cdot, \cdot) * g_\sigma(I(x)))$$

where $\partial_1 \varphi(\cdot, \cdot)$ is the partial derivative of $\varphi(\cdot, \cdot)$ according to the first variable, $\partial_1 \varphi(\hat{q}(\cdot, \cdot) * g_\sigma)$ denotes the convolution of the function $\partial_1 \varphi(\hat{q}(\cdot, \Omega) : \mathcal{R}^m \rightarrow \mathcal{R}$ with the function $g_\sigma$, and:

$$C(\Omega) = \int_{\mathcal{R}^m} \partial_1 \varphi(q(\alpha, \Omega), \alpha) q(\alpha, \Omega) d\alpha$$

An experimental result is given for matching regions in video sequences. The level set method introduced by [14] and fast smoothing B-splines [15] have been tested to implement the evolution equation. In this experiment, we use both the distance between the reference histogram of the background and the reference histogram of the region and also a regularization term minimizing the curve length. We choose the HSV space color and we work on 2D-histograms with $H$ and $V$, so $m = 2$. Results are given for sequence “Erik”. The reference segmentation is given in Fig.1 with the corresponding region histogram. This reference is used to make the contour evolve on another image of the sequence, Fig.2. The region histogram evolves towards the reference histogram and the face is well segmented.
we define a criterion combining geometric prior, statistical measures computed on object region, and a regularization term:

$$J(\Omega) = \int_\Gamma \varphi(d(\Gamma(s), \Gamma^{ref}))ds + \lambda \int_\Omega k_r(x, \Omega)dx + \epsilon \int_\Gamma ds.$$  

(5)

The first term of criterion (5) is the geometric prior. Let $\Gamma^{ref}$ denotes the reference contour. The reference contour may be initialized from an atlas for registration applications, or it may be defined interactively by an operator. The geometric term minimizes $d(\Gamma, \Gamma^{ref})$, the geometric distance $d$ between contours $\Gamma$ and $\Gamma^{ref}$. For each $x \in \Gamma$, $d(x, \Gamma^{ref}) = \min_{y^{ref} \in \Gamma^{ref}} (|x - y^{ref}|) = |x - y(x)|$ (see Fig. 3.a). $\varphi$ is a differentiable function. The deformation between the reference shape and the active contour is a free form deformation as opposed to a constrained parametric transformation [20]. The first term is differentiated in details in [21] and [12].

The second term of criterion (5) characterizes the object region, for instance in terms of color texture or motion. For homogeneous region segmentation, $k_r(x, \Omega)$ may be a statistical descriptor such as the mean, the variance, the determinant of covariance matrix [9] or the region histogram [11]. $\psi^{in}$ is a differentiable function. The third term includes a smoothness constraint by minimizing the length of the contour.

Differentiation of each term of the criterion leads to the evolution equation of the active contour:

$$\frac{\partial \Gamma(\tau)}{\partial \tau} = \left[ (N^{ref}, N) \varphi'(d) + \varphi(d) \kappa + \lambda k_r(x, \Omega) + A(x, \Omega) + \epsilon \kappa \right] N.$$  

(6)

The proposed free form deformation algorithm was applied to interactive image segmentation based on a competition between a reference contour constraint and statistical region-based features of the object region to be segmented. For this application, we chose $\varphi(d) = \sqrt{1 + d^2} - 1$ [22]. Descriptor $k_r(x, \Omega)$ is chosen as the sum of variances of the object region as presented in Section 2.1.

The purpose of this application was to evaluate the benefits of the geometrical prior compared to an unconstrained region-based segmentation. Figure 3 shows the results of the unconstrained (Fig. 3.b) and geometrically constrained (Fig. 3.d) segmentation methods. Figure 3.c shows the reference contour as defined by an operator. Using the unconstrained segmentation, the contour drifts from the face towards the hand. This drift is overcome by the geometric prior based upon the distance between the active contour and the reference contour.

### 3. COMBINING GEOMETRIC PRIOR AND STATISTICAL DESCRIPTOR

Several approaches introduce shape prior information in active contour [16, 17, 18, 19]. For interactive segmentation,
4. CONCLUSION

In this paper, region-based active contours are used for image and video segmentation. The evolution equation of the active contour is computed for the minimization of functionals including statistical region-dependent features and geometrical boundary-dependent features. To illustrate, a statistical descriptor using the distance between the region histogram and a reference one is performed for matching regions in video sequences. Besides, statistical and geometrical features are efficiently combined for segmentation as shown in the experimental results. Further research may be concerned with the evaluation of other combinations taking benefit of both geometrical and statistical features [21].

5. REFERENCES


