SHAPE GRADIENT FOR IMAGE SEGMENTATION USING INFORMATION THEORY

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Goal:
- Using information theory as criterion for segmentation

Approach:
- Characterization of the region using information theory
  - $\Omega_{opt} \Rightarrow \text{Min of a functional } J(\Omega)$
- Research of an optimum using active contour deformation
  - $\Gamma(\tau) \rightarrow \Omega_{opt}$
Segmentation using active contours

- Problems:
  - How to characterize the region?
  - How to compute the evolution equation of the active contour?
Characterization of the region

- Homogeneous features of the region
  - Mean, variance, covariance matrix …

- Problem:
  - Assume Gaussian model of distributions

- Idea:
  - Criterion based on real distributions => information theory
- Histograms

- Given a region $\Omega$, the histogram of this region is:

$$q(I(x), \Omega) = \Pr \{ I(x) | x \in \Omega \}$$

- Histograms are estimated using the Parzen window method

$$q(I(x), \Omega) = \frac{1}{|\Omega|} \int_{\Omega} K(I(x) - I(\tilde{x}))d\tilde{x}$$

where $K$ is the Gaussian kernel of this estimation.
A general criterion is defined as follows:

\[ J(\Omega) = \int_{\Omega} \varphi(q(I(x),\Omega))dx + \int_{\Gamma} \lambda ds \]

Region-based term: \( J_r(\Omega) \)

Boundary-based term: \( J_b(\Omega) \)

with \( \varphi \) a function: \( \mathbb{R}^+ \rightarrow \mathbb{R} \) of the probability \( q \) and \( \lambda \) a regularization parameter.
Homogeneity criterion: entropy

- Given a region $\Omega$, the function $\varphi$ is the following:

$$\varphi(q(I(x), \Omega)) = -q(I(x), \Omega) \ln q(I(x), \Omega)$$

- And the entropy of the region $\Omega$:

$$H(\Omega) = \int_{\Omega} -q(I(x), \Omega) \ln q(I(x), \Omega) dx$$

[Herbulot & al. «Shape gradient for image segmentation using information theory», ICASSP 2004]
Homogeneity criterion for color images: joint entropy

Definition of joint entropy:

\[ H_{XY}(\Omega) = -\int_{\Omega} q(I_x(x), I_y(x), \Omega) \ln q(I_x(x), I_y(x), \Omega) dx \]

where \( I_x \) and \( I_y \) are two channels of the image

[Herbulot & al. «Shape gradient for multimodal image segmentation using joint intensity distributions», WIAMIS 2004]
Derivation tool

- Derivation using shape gradient method:

[Jehan & al. «DREAM2S: Deformable Regions driven by an Eulerian Accurate Minimization Method for image and video segmentation », IJCV 2003]

\[
\begin{align*}
\Omega & \rightarrow \Omega(\tau)
\end{align*}
\]

Dynamic scheme

\[
dJ_r(\Omega, V) = \lim_{\tau \rightarrow 0} \frac{(J_r(\Omega(\tau)) - J_r(\Omega))}{\tau}
\]

Eulerian derivative

\[
dJ_r(\Omega, V) = -\int_{\Gamma} \varphi(q(I(x), \Omega))(V \cdot N)dx + \int_{\Omega} \varphi'(q(I(x), \Omega), V)dx
\]

Evolution equation

\[
\frac{\partial \Gamma}{\partial \tau}(x) = (\varphi(q(I(x), \Omega)) + A(x, \Omega))N
\]
Evolution equation

- Evolution equation for a region $\Omega$ in the case of entropy:

$$\frac{\partial \Gamma}{\partial \tau}(x) = [-q(I(x), \Omega)(\ln q(I(x), \Omega) + 1) - \frac{1}{|\Omega|}[H(\Omega) - 1$$

$$+ \int_{\Omega} K(I(\vec{x}) - I(x)) \ln q(I(\vec{x}), \Omega) d\vec{x}]][N$$

- Region competition:

$$J(\Omega_{in}, \Omega_{out}, \Gamma) = H(\Omega_{in}) + H(\Omega_{out}) + \int_{\Gamma} \lambda ds$$
Implementation

- Use of cubic B-splines
- 😞
  - No automatic topology change (unlike level-set methods)
- 😊
  - Low processing cost, real time video processing
Minimization of joint entropy

- On color images, channels are Y and U:

Iteration 0

Iteration 100
Minimization of joint entropy

Iteration 300

Iteration 500
Conclusion

- Entropy is a relevant descriptor for image and video segmentation


- **References:**
  
  - A. Herbulot, S. Jehan-Besson, M. Barlaud, G, Aubert “Shape gradient for multimodal image segmentation using joint intensity distributions”, 5th international Workshop on Image Analysis for Multimedia International Services, 2004